Extra Credit Rocks

Sign up for a Discover® Student Card today and enjoy:

- 0% Intro APR* on Purchases for 6 Months
- No Annual Fee
- Easiest Online Account Management Options
- Full 5% *Cashback Bonus*®* on Get More purchases in popular categories all year
- Up to 1% *Cashback Bonus*®* on all your other purchases
- Unlimited cash rewards that never expire as long as you use your Card

APPLY NOW

DISCOVER®
CARD

*View Discover® Card Rates, Fees, Rewards and Other Important Information.

Student's Solutions Manual

to accompany

College Algebra and Trignometry with Applications

Second Edition

Wesner • Mahler

Prepared by

Philip H. Mahler

Middlesex Community College

Campfire queen Cycling champion Sentimental geologist*

Learn more about Marjon Walrod and tell us more about you. Visit pwc.com/bringit.

Your life. You can bring it with you.

*connectedthinking

PRICEWATERHOUSECOOPERS

GetMath Educational Software http://www.getmath.com

ISBN 1-932661-72-7

Printed in the United States of America by Bernard J. Klein Publishing

10 9 8 7 6 5 4

Contents


College Loan
CLC
Corporation
Need more money for college expenses?
The CLC Private Loan℠ can get you up to $40,000 a year for college-related expenses.
Here's why the CLC Private Loan℠ is a smart choice:
Approved borrowers are sent a check within four business days
Get $1000 - $40,000 each year
Interest rates as low as prime + 0% (8.66% APR)
Quick and easy approval process
No payments until after graduating or leaving school
CLICK HERE
or call 800.311.9615.
We are available 24 hours a day, 7 days a week.

Student Solutions Manual to Accompany College Algebra and Trigonometry with Applications

Introduction to the Student

When you study mathematics you are doing something which will help you for the rest of your life. Although most students do not realize it mathematics is used in almost every discipline which a student is likely to enter, from nuclear physics to music. In fact, in the age of electronic calculating devices mathematics is more important than ever. Consider the following survey of a few areas of specialization and just a few of the ways in which mathematics applies to them. The truth is that there is virtually no area of study which is not touched by sophisticated mathematical principles.

Astronomy — Some of higher mathematics like trigonometry and calculus was created just to describe the laws of nature which govern the motion of the planets. Today astronomers who work on the theoretical side of this discipline talk about black holes and superstrings, among other things. So far these are things which no one has ever seen, but seem to be predicted by the laws of nature, and they are expressed in the language of mathematics.

Aviation — The principles of navigating an aircraft use trigonometry. The weight and balance of an aircraft are critical to the safety of the flight, and are studied with formulas, graphs and charts. The theory behind electronic systems of navigation in which position is located with reference to satellites is entirely dependent upon mathematical theory.

Biology — The laws of population growth and inheritance are mathematical. The way in which an organism's size governs its weight, the way in which its surface area governs its ability to breath (in the case of insects) or to pass nutrients, etc. (in the case of microscopic organisms) are all described mathematically. The way in which the lungs pass oxygen, or the heart pumps blood, are all described using mathematics.

Business — Accounting principles are stated using mathematical formulas. Forecasting profits, the future value of money, break even points, and many, many other things, are described mathematically. A whole area of mathematics called linear programming was created to serve the needs of large industries to efficiently allocate resources.

Chemistry — Balancing equations which describe reactions, moles, the shape of crystals, the heat given off in a reaction are just some of the mathematically described phenomena which are evident when leafing through any text on chemistry.

Economics — This discipline creates mathematical models which use are quite complicated. These models are used to study economies of cities, nations, and the world, and to forecast future economic trends. Studies of economies extensively use statistics. To do graduate work in this field one needs a great deal of mathematics.

Ecology The rate at which toxins accumulate in the environment, the water cycle, the study of the ozone layer and the greenhouse effect are all heavily reliant on mathematics. The probable number of leaking underground oil storage tanks, or the amount of gasoline vapors passing into the air, are more examples of places in which mathematics would be applied.

Electronics Ohm's Law, Kirchoff's Law, phase diagrams, Lissajous patterns, and the logarithmic response of a transistor's output in a certain circuit are all examples which underline the fact that mathematics is the language of electronics.

Literature Some people say that Christopher Marlowe and others wrote much of the work which is attributed to Shakespeare. This question has been studied using mathematics! In particular, quantitative, statistical comparisons of an author's known writings with other writings have been used to help answer this kind of controversy.

History Many historians use historical data to help study history. For example, studying the amount of exports and imports of different raw materials and manufactured goods over time can tell something about the economy of a culture in the past.

Law A great deal of mathematics is now used in the courts in lawsuits over pollution, paternity, forensics, ballistics, consumer fraud etc. Drug testing uses statistical principles. There is a lot of mathematics on tests used to gain admission to law school.

Medicine Things like the transmission of a disease through a population, the percentage of a population which must be inoculated to control a certain disease, or the efficiency of a certain dose of a medicine are all described mathematically. Heart rhythms, breathing and blood flow are studied mathematically. The most efficient distribution of limited medical resources needs mathematics to be studied. In sports medicine the mathematical principles of physics are applied - for example, the arm is a lever, and the elasticity of a bone is a physical phenomenon which can be described with an equation.

Music The principles of tone and harmony are mathematical. The modern 12-tone "equally tempered" musical scale can only be properly explained mathematically. Joseph Schillinger used mathematical principles in the 1930's to help create music, and taught them to George Gershwin before Gershwin wrote his opera *Porgy and Bess*.

Physics At one time astronomy, physics, and mathematics were one discipline. Anyone who has studied any physics understands how thoroughly physics uses mathematics to describe physical phenomena. Indeed, modern theoretical physics uses mathematics, and not laboratory experiments, to discover new facts of nature. (To be sure, only experimentation can confirm these discoveries.)

Psychology Much research in psychology is statistical in nature, which is thoroughly mathematical. Indeed, there is a journal called *The Journal of Mathematical Psychology*! Powerful computers are also used to model the synapses of the brain to attempt to discover the principles of this organ, perhaps the single most amazing thing in nature. The models postulated for the structure of the brain are highly mathematical.

Sociology As in psychology and other social sciences statistical research is used extensively. The determination of whether, say, being traumatized when young leads to

pathological social behavior is a question which requires research, insight, and *mathematics* to answer.

Assumptions upon which the text was created

In writing this text we have assumed that you have completed an intermediate algebra course, and therefore has been introduced to solving equations, factoring, radicals and graphing linear equations. This means that the language of algebra should not be new to you. The following problems and their solutions should not seem entirely foreign to you, even if you have forgotten some of the details.

1. Solve the equation $5x - 3 = 2(x + 7)$

Solution:		
	$5x - 3 = 2x + 14$	
	$5x - 2x - 3 = 14$	Subtract $2x$ from each member
	$3x - 3 = 14$	$5x - 2x = 3x$
	$3x = 17$	Add 3 to both members
	$x = \frac{17}{3}$	Divide each member by 3

2. Factor $3x^2 - 2x - 4$

Solution:		
	$3x^2 - 9x - 4$	
	$(3x \quad)(x \quad)$	$3x \cdot x = 3x^2$
	$(3x \pm 4)(3x \pm 1)$	Several ways to get -4 in the third term
	$(3x \pm 2)(3x \pm 2)$	
	$(3x - 4)(3x + 1)$	The choice which gives $-9x$ for the middle term

3. Simplify $\sqrt{8x^5}$

Solution:		
	$\sqrt{8x^5} = \sqrt{2^2 \cdot 2 \cdot x^4 \cdot x}$	Factor
	$= 2x^2\sqrt{2x}$	$\sqrt{2^2} = 2; \sqrt{x^4} = x^2$

It is also assumed that you own a scientific calculator or graphing calculator, and are familiar with the basic keys for arithmetic computation. Keystrokes for a typical scientific calculator are presented in the text where they go beyond the basic arithmetic operations.

Graphing Calculators/Computers

You may own a graphing calculator instead of a scientific calculator. The text specifically indicates how to use these devices. Examples are presented based on the TEXAS INSTRUMENTS TI–81 graphing calculator. The introductory section *Computer Aided Mathematics* introduces the basic principles involved in using a graphing calculator, using the TI–81 as an example.

Tips for Success

- **Work** Success in a mathematics course requires a lot of work! You must work assigned problems all the time. Much of what you will learn is complicated and needs constant practice.
- **Regular study time** Try to set aside some time every day in which you will do some mathematics. If you have already finished all assigned problems then go back to previous sections of the text and do some review problems. Certain sections will have seemed harder than others. Go back to these harder sections! With enough work you will understand everything. Remember what Thomas Edison said. "Genius is 10% inspiration and 90% perspiration."
- **You can do it!** It is true that some people have a "knack" for mathematics, just as some do for tennis, basketball, music, drawing, or English composition. However, this does not mean that you cannot do these things if you don't have the knack – it just means that you have to work at it. No one gets good at anything without working at it. If they don't seem to work at something it's just because they love it and don't consider what they do to be work.
- **How to study** Read the text seated at a desk. Have a pencil and paper at hand. When you come to an example, copy the steps onto your paper as you read. This will help you understand better for several reasons.

  – Mathematics looks different when it is written by hand than when it is printed. You should get used to what it looks like when you write it!

  –Some ways are better than others to organize mathematics problems. We try to show you a good way to do this in the text, and you want to get used to it.

  –It has been shown that one learns best when more than one sense is used. Writing uses motor skills as well as sight.
- **Do the homework** It has been said that mathematics is not a spectator sport. You could watch someone play the piano for years, but this would not help you learn to play the piano. You must do it yourself.
- **Mathematics Ability is not in the Genes** – *Anyone can do mathematics*. Many people tend to feel that difficulties in learning mathematics indicate lack of ability, and that mathematics should therefore be avoided by an individual encountering problems. The answer to difficulties is to work harder, not to give up. Mathematics is very important throughout most professions, and it just won't go away!

Features of the text to help you learn

Problem Sets

- The drill portion of the problem sets are similar to the examples of that section.
- The *answers to all problems* appear at the end of the text, except for even–numbered exercises. The *complete solutions to selected problems*, indicated by boxing the problem number, also appear at the end of the text. If you have trouble with a particular problem you should be able to find the solution to a similar problem either in the examples in the text, the selected (box numbered) problems, or in this manual.
- Each problem set ends with *Skill and Review* problems. These either review old material or prepare for the next section. The *solutions* to these problems appear in appendix B. You should always work these problems.
- The exercises progress from straightforward application of the material covered in the exposition to problem solving via more difficult application problems and then to problems which require some ingenuity and creativity. Those problems requiring exceptional ingenuity or amount of work are marked with the symbol . If you try the complete range of problems you will have a good introduction to the way in which this material is used in a variety of disciplines.

Chapter Summary, Review, Test at the end of each chapter

- Chapter Summary – This reviews the highlights and key points of the chapter. Read this over after the chapter is completed. Think about each item. If something seems unfamiliar go and find it in the text and review it.
- Chapter Review – This presents review problems from the chapter, keyed to sections. You should work this after the chapter is completed.
- Chapter Test – This is designed to help you practice the material as it might appear on a test, out of the context of each section. The chapter test may well be longer than what would be given in class. The test provides material out of the context provided by knowing what section it is from, and by being surrounded by similar problems. In the homework sets there are inevitably many clues to the method of solution, including nearby problems and temporal and physical proximity to explanations. The chapter test is an aid to make that last link in learning – recognition of problem type, with attending method of solution.

Applications — Applications problems are put in the text to show you where mathematics is used in the various disciplines. Do not be afraid of them. You will see that you don't have to know, for example, electronics to do those applications which apply to electronics. The problems always make clear what mathematical operations and principles are involved.


Love The Taste.
Taste The Love.
At Culver's® we can't think of anything better than serving up our creamy frozen custard and delicious classics cooked fresh the minute you order them. Which is why when we bring them to your table, they're always accompanied by a warm smile and a friendly offer to see if there's anything else we can get for you. So come on into your neighborhood Culver's and see for yourself. You might just be in love by the time you leave.
Culver's® FROZEN CUSTARD BUTTERBURGERS
Culver's FROZEN CUSTARD BUTTERBURGERS
© 2005 Culver Franchising System, Inc.
www.culvers.com

Chapter 1

Exercise 1–1

1. $\{4, 5, 6, 7, 8, 9, 10, 11\}$

5. $\{\ 3(-2), 3(-1), 3(0), 3(1), 3(2), 3(3), 3(4)\ \}$
$\{\ -6, -3, 0, 3, 6, 9, 12\ \}$

9. $0.230769\ 230769$ Repeating

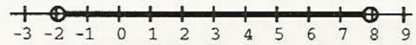
13. $\frac{5}{8} - \frac{3}{5} + \frac{3}{4}$
$\frac{5(5) - 3(8)}{8(5)} + \frac{3}{4}$
$\frac{1}{40} + \frac{3 \cdot 10}{4 \cdot 10}$
$\frac{31}{40}$

17. $\frac{-3(5 - 2(9 - 12) + 4) - (8 - 2)(2 - 8)}{5(3 - 7)^2 - (3 - 7)^3}$
$\frac{-3(5 - 2(-3) + 4) - (6)(-6)}{5(-4)^2 - (-4)^3}$
$\frac{-3(5 - (-6) + 4) - (-36)}{5(16) - (-64)}$
$\frac{-3(5 + 6 + 4) + 36}{80 + 64} = \frac{-3(15) + 36}{144}$
$\frac{-45 + 36}{144} = \frac{-9}{144} = -\frac{1}{16}$

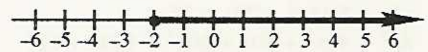
In problem 21 use the pattern $\frac{a}{c} \pm \frac{b}{d} = \frac{ad \pm bc}{cd}$

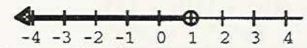
21. $\frac{x - y}{4x} - \frac{2x + y}{3y}$
$\frac{3y(x - y) - 4x(2x + y)}{4x(3y)}$
$\frac{3xy - 3y^2 - 8x^2 - 4xy}{12xy}$
$\frac{-xy - 3y^2 - 8x^2}{12xy}$

25. $\frac{3x}{2y} \div \frac{5y}{2x} \cdot \frac{x}{5y}$
$\frac{3x}{2y} \cdot \frac{2x}{5y} \cdot \frac{x}{5y}$ To divide, invert and multiply
$\frac{6x^3}{50y^3}$
$\frac{3x^3}{25y^3}$

29. $(-2, 8)$ 

33. $(-\sqrt{2}, \pi]$ 
$\sqrt{2} \approx 1.4$

37. $[-2, \infty)$ 

41. $\{\ x \mid x < 1\ \}$ 

45. $\{\ x \mid -\frac{1}{2} < x \le 1\frac{1}{2}\ \}, (-\frac{1}{2}, 1\frac{1}{2}]$

49. $\{\ x \mid -2 < x < \frac{1}{2}\ \}, (-2, \frac{1}{2})$

53. $|-4|$
$-(-4)$
4

57. $|-\sqrt{10} - 3|$
$-(-\sqrt{10} - 3)$
$\sqrt{10} + 3$

61. $|(-5)^2|$
$|25|$
25

65. $|2x^4|$
$2x^4 \qquad 2x^4 > 0$

69. $-|-5x^2|$
$-(\ |-5| \cdot |x^2|\ )$
$-(5x^2)$
$-5x^2$

73. $|(x - 2)^2(x + 1)|$
$|(x - 2)^2| \cdot |x + 1|$
$(x - 2)^2\,|x + 1|$

77. $\frac{-8 - 6 + 5 + 2 + 4}{5} = \frac{-3}{5} = -\frac{3}{5}$ or -0.6

Exercise 1-2

1. $2x^5 \cdot x^2 \cdot x^4$
$2x^{5+2+4}$ To multiply like bases add exponents
$2x^{11}$

5. $(3a^5b)(2a^2b^2)$
$6a^{5+2}b^{1+2}$
$6a^7b^3$

9. $3x^4y^{-3}$
$3x^4\left(\frac{1}{y^3}\right)$
$\frac{3x^4}{y^3}$

13. $(-3^2a^2b^{-3})^2$ To raise a term to a power multiply the power and each exponent
$(-9a^2b^{-3})^2$
$(-9)^2a^4b^{-6}$
$\frac{81a^4}{b^6}$

17. $(-3)^{-3}$
$\frac{1}{(-3)^3}$ Negative exponents indicate fractions
$-\frac{1}{27}$

21. $(2x^4y)(-3x^3y^{-2})$
$-6x^7y^{-1}$
$-\frac{6x^7}{y}$

25. $\frac{3^2x^{-4}y}{3xy^{-4}}$
$\frac{9y \cdot y^4}{3x \cdot x^4}$
$\frac{3y^5}{x^5}$

29. $\left(\frac{-2x^2y}{5x^{-3}y^3}\right)^3$
$-\frac{2^3x^6y^3}{5^3x^{-9}y^9}$
$-\frac{8x^{15}}{125y^6}$

33. $\left(\frac{3x^{-2}}{12x^{-4}}\right)\left(\frac{16x^{-5}}{2x}\right)$
$\frac{x^2}{4} \cdot \frac{8}{x^6}$
$\frac{2}{x^4}$

37. $x^{2n-3}\,x^{2n+3}$
$x^{(2n-3)+(2n+3)}$
x^{4n}

41. $\left(\frac{x^ny^{2-n}}{x^{-n}y^{n-2}}\right)^4$
$\left(\frac{x^nx^n}{y^{n-2}y^{-(2-n)}}\right)^4$
$\left(\frac{x^{2n}}{y^{2n-4}}\right)^4$
$\frac{x^{8n}}{y^{8n-16}}$

45. $-19{,}002{,}000{,}000{,}000$
-1.9002×10^{13}

49. $0.000\ 000\ 000\ 003\ 502$
3.502×10^{-12}

53. -1.384×10^{-10}
$-0.000\ 000\ 000\ 138\ 4$

57. 9.1×10^{-28} g

61. polynomial, degree 3

65. $3\sqrt{x} - 4x - 2$ not a polyomial because of the $\sqrt{x}$.

69. $\frac{1}{3}a - 2cd^2 + \sqrt{b}$
$\frac{1}{3}(-5) - 2(-\frac{1}{2})(6)^2 + \sqrt{4}$
$-\frac{5}{3} + 1(36) + 2$
$-1\frac{2}{3} + 38$
$-\frac{2}{3} + 37$
$36\frac{1}{3}$

73. $(5x^2 - 3x - 2) - (3x^2 + x - 8)$
$5x^2 - 3x - 2 - 3x^2 - x + 8$
$5x^2 - 3x^2 - 3x - x - 2 + 8$
$2x^2 - 4x + 6$

77. $(3x^2y - 2xy^2 - xy) + (2xy^2 - 5x^2y + 3xy)$
$3x^2y - 2xy^2 - xy + 2xy^2 - 5x^2y + 3xy$
$3x^2y - 5x^2y - 2xy^2 + 2xy^2 - xy + 3xy$
$-2x^2y + 2xy$

81. $[-(3a - b) - (2a + 3b)] - [(a - 6b) - (3b - 10a)]$
$[-3a + b - 2a - 3b] - [a - 6b - 3b + 10a]$
$[-5a - 2b] - [11a - 9b]$
$-5a - 2b - 11a + 9b$
$-16a + 7b$

85. $-2a^2b(5a^2b + 3a^2b^2 - 2ab^3)$
$-10a^4b^2 - 6a^4b^3 + 4a^3b^4$

89. $(3x+y)(5x-y)$
$15x^2-3xy+5xy-y^2$
$15x^2+2xy-y^2$

93. $(2x^2-3x+1)(5x^2-2x+7)$
$10x^4-4x^3+14x^2-15x^3+6x^2-21x+5x^2-2x+7$
$10x^4-19x^3+25x^2-23x+7$

97. $(x+2y)(x-3y)(x+y)$
$(x+2y)(x^2-2xy-3y^2)$
$x^3-2x^2y-3xy^2+2x^2y-4xy^2-6y^3$
$x^3-7xy^2-6y^3$

101. $(2a-3)(3a-2b+c)$
$6a^2-4ab+2ac-9a+6b-3c$

105. $(2x+5)^3$
$(2x+5)(2x+5)(2x+5)$
$(2x+5)(4x^2+20x+25)$
$8x^3+40x^2+50x+20x^2+100x+125$
$8x^3+60x^2+150x+125$

109. $\dfrac{6a^6b^2-8a^4b^4+12a^4b^6}{2a^4b^2}$
$=\dfrac{6a^6b^2}{2a^4b^2}-\dfrac{8a^4b^4}{2a^4b^2}+\dfrac{12a^4b^6}{2a^4b^2}=3a^2-4b^2+6b^4$

113. $\dfrac{x^2-3x+2}{x-1}=x-2$

$$\begin{array}{r|l} & x-2 \\ \hline x-1 & x^2-3x+2 \\ & x^2-x \\ \hline & -2x+2 \\ & -2x+2 \\ \hline & 0 \end{array}$$

117. $\dfrac{6x^3+x^2-10x+5}{2x+3}=3x^2-4x+1+\dfrac{2}{2x+3}$

$$\begin{array}{r|l} & 3x^2-4x+1 \\ \hline 2x+3 & 6x^3+x^2-10x+5 \\ & 6x^3+9x^2 \\ \hline & -8x^2-10x+5 \\ & -8x^2-12x \\ \hline & 2x+5 \\ & 2x+3 \\ \hline & 2 \end{array}$$

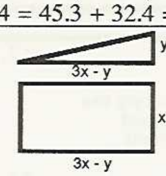
121. $\dfrac{4x^3-x^2+5}{x^2-x+1}=4x+3+\dfrac{-x+2}{x^2-x+1}$

$$\begin{array}{r|l} & 4x+3 \\ \hline x^2-x+1 & 4x^3-x^2+0x+5 \\ & 4x^3-4x^2+4x \\ \hline & 3x^2-4x+5 \\ & 3x^2-3x+3 \\ \hline & -x+2 \end{array}$$

125. (a) $(3t_1+4t_2)-(t_1+6t_2-3t_3)$
$3t_1+4t_2-t_1-6t_2+3t_3$
$3t_1-t_1+4t_2-6t_2+3t_3$
$2t_1-2t_2+3t_3$
(b) $(3t_1+4t_2)(t_1+t_2-3t_3)$
$3t_1^2+3t_1t_2-9t_1t_3+4t_1t_2+4t_2^2-12t_2t_3$
$3t_1^2+7t_1t_2-9t_1t_3+4t_2^2-12t_2t_3$
(c) $(-3x_1^2x_2^4)(2x_1x_2)^3$
$(-3x_1^2x_2^4)(8x_1^3x_2^3)$
$-24x_1^5x_2^7$

129. $G=0.15(T_1+T_2+T_3+T_4)+0.4E$
$G=0.15(68+78+82+74)+0.4(81)$
$=0.15(302)+32.4=45.3+32.4=77.7$

133. The area is the area of the triangle plus the area of the rectangle.
Area = Triangle + Rectangle $=\frac{1}{2}(3x-y)(y)+(3x-y)(x)=\frac{1}{2}(3xy-y^2)+(3x^2-xy)$
$=\frac{3}{2}xy-\frac{1}{2}y^2+3x^2-xy=3x^2+\frac{1}{2}xy-\frac{1}{2}y^2$



137. (a) $(a+b)(c+d)$: 7 $\quad ac+ad+bc+bd$: 23
(b) $(2a+3b)(a-2b)$: 22 $\quad 2aa-ab-6bb$: 27
(c) $a(b+c-d)$: 7 $\quad ab+ac-ad$: 17
(d) $(a+b)(c+d+e)$: 8 $\quad ac+ad+ae+bc+bd+be$: 35
(e) $(a+b)(c+d)(e+f)$: 13
$ace+ade+bce+bde+acf+adf+bcf+bdf$: 87

141. $\sqrt{4{,}000{,}000{,}000{,}000{,}000{,}000}$
$\sqrt{4 \times 10^{18}}$
2×10^9
4 $\boxed{\text{EXP}}$ 18 $\boxed{\sqrt{x}}$
$\boxed{\text{TI-81}}$: $\boxed{\text{2nd}}$ $\boxed{\sqrt{\ }}$ 4 $\boxed{\text{EE}}$ 18 $\boxed{\text{ENTER}}$

Exercise 1–3

See the factoring flow chart on the next page.

1. $3(4x^2-3xy-6)$

5. $(a-b)(6x+5y)$

9. $2m(n+5)-1(n+5)-p(n+5)$
$(n+5)[2m(\cancel{n+5})-1(\cancel{n+5})-p(\cancel{n+5})]$
$(n+5)(2m-1-p)$

13. $5ax+15bx-ay-3by$
$5x(a+3b)-y(a+3b)$
$(a+3b)(5x-y)$

17. $(x+4y)(x+3y)$

21. $x^2-18x+32$
$(x-2)(x-16)$

25. x^4-16y^4
$(x^2-4y^2)(x^2+4y^2)$
$(x-2y)(x+2y)(x^2+4y^2)$

29. $(2a+5)((2a)^2-5(2a)+5^2)$
$(2a+5)(4a^2-10a+25)$

33. $(y-2)^2+5(y-2)-36$
$z^2+5z-36$ $\quad$ Replace $y-2$ by z
$(z+9)(z-4)$
$[(y-2)+9][(y-2)-4]$ $\quad$ Replace z by $y-2$
$(y+7)(y-6)$

37. $(2x-y)^2-(x+y)^2$
a^2-b^2 $\quad$ Replace $2x-y$ by z, $x+y$ by b
$(a-b)(a+b)$
$[(2x-y)-(x+y)][(2x-y)+(x+y)]$ $\quad$ Replace z by $2x-y$, b by $x+y$
$(x-2y)(3x)$

41. $16x^{12} + 2x^3$
$2x^3(8x^9 + 1)$
$2x^3(2x^3 + 1)(4x^6 + 2x^3 + 1)$

45. $(m - 7)(m + 7)$

49. $(7a + 1)(a + 5)$

53. $(ab + 4)(ab - 2)$

57. $25x^2(3x + y) + 5x(3x + y)$
$(3x + y)(25x^2 + 5x)$
$5x(3x + y)(5x + 1)$

61. $4m^2 - 16n^2$
$4(m^2 - 4n^2)$
$4(m - 2n)(m + 2n)$

65. $3x^6 - 81y^3$
$3(x^6 - 9y^3)$
$3(x^2 - 3y)(x^4 + 3x^2y + 9y^2)$

69. $4x^2 - 36y^2$
$4(x^2 - 9y^2)$
$4(x - 3y)(x + 3y)$

73. $27a^9 - b^3c^3$
$(3a^3 - bc)(9a^6 + 3a^3bc + b^2c^2)$

77. $a^4 - 5a^2 + 4$
$(a^2 - 4)(a^2 - 1)$
$(a - 2)(a + 2)(a - 1)(a + 1)$

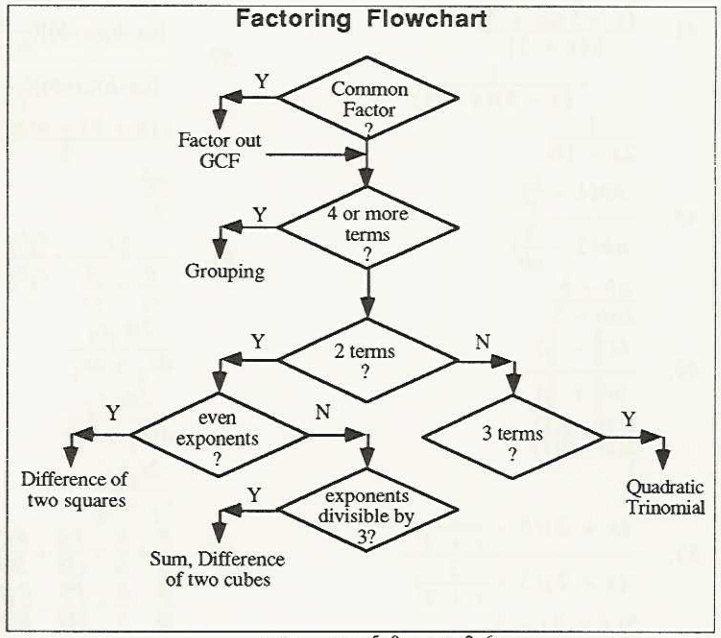
81. $y^4 - 16$
$(y^2 - 4)(y^2 + 4)$
$(y - 2)(y + 2)(y^2 + 4)$

85. $(x + y)^2 - 8(x + y) - 9$
$z^2 - 8z - 9 \qquad z = x + y$
$(z + 1)(z - 9)$
$(x + y + 1)(x + y - 9)$

89. $4ab(x + 3y) - 8a^2b^2(x + 3y)$
$(x + 3y)(4ab - 8a^2b^2)$
$4ab(x + 3y)(1 - 2ab)$

93. $80x^5 - 5x$
$5x(16x^4 - 1)$
$5x(4x^2 - 1)(4x^2 + 1)$
$5x(2x - 1)(2x + 1)(4x^2 + 1)$

97. $9a^2 - (x + 5y)^2$
$9a^2 - b^2 \qquad b = x + 5y$
$(3a - b)(3a + b)$
$[3a - (x + 5y)][3a + (x + 5y)]$
$(3a - x - 5y)(3a + x + 5y)$



101. $24x^5y^9 + 81x^2z^6$
$3x^2(8x^3y^9 + 27z^6)$
$3x^2(2xy^3 + 3z^2)(4x^2y^6 - 6xy^3z^2 + 9z^4)$

105. $\pi r_2^2 - \pi r_1^2$
$\pi(r_2^2 - r_1^2)$
$\pi(r_2 - r_1)(r_2 + r_1)$

109. As a difference of two squares:
$x^6 - 1 = (x^3 - 1)(x^3 + 1)$
$= (x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$
$= (x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)$

As a difference of two cubes:
$x^6 - 1 = (x^2 - 1)(x^4 + x^2 + 1)$
$= (x - 1)(x + 1)(x^4 + x^2 + 1)$

Thus, $(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1) =$
$(x - 1)(x + 1)(x^4 + x^2 + 1)$, so $x^4 + x^2 + 1$ must be the same as $(x^2 + x + 1)(x^2 - x + 1)$.

Exercise 1–4

1. $\dfrac{24\,p^5\,q^7}{18\,p\,q^3}$
$\dfrac{4p^4q^4}{3}$

5. $\dfrac{(a - 3)(a + 3)}{4(a + 3)}$
$\dfrac{a - 3}{4}$

9. $\dfrac{6(a - b)(a^2 + ab + b^2)}{(a - b)(a + b)}$
$\dfrac{6(a^2 + ab + b^2)}{a + b}$

13. $\dfrac{(a + 6)(3a - 2)}{-(3a^2 + 7a - 6)}$
$\dfrac{(a + 6)(3a - 2)}{-(a + 3)(3a - 2)}$
$-\dfrac{a + 6}{a + 3}$

17. $\dfrac{x(x - 1) - (x + 1)(3 - x)}{(x + 1)(x - 1)}$
$\dfrac{x^2 - x - (-x^2 + 2x + 3)}{x^2 - 1}$
$\dfrac{x^2 - x + x^2 - 2x - 3}{x^2 - 1}$
$\dfrac{2x^2 - 3x - 3}{x^2 - 1}$

21. $\dfrac{3a - 10}{5a} + \dfrac{9}{2b}$
$\dfrac{2b(3a - 10) + 9(5a)}{5a(2b)}$
$\dfrac{45a + 6ab - 20b}{10ab}$

25. $\dfrac{2}{x - 3} + \dfrac{5}{x(x - 3)}$
$\dfrac{2x}{x(x - 3)} + \dfrac{5}{x(x - 3)}$
$\dfrac{2x + 5}{x(x - 3)}$

29. $\dfrac{2a - 1}{2a - 3} + \dfrac{4 - a}{-(2a - 3)}$
$\dfrac{2a - 1}{2a - 3} - \dfrac{4 - a}{2a - 3}$
$\dfrac{(2a - 1) - (4 - a)}{2a - 3}$
$\dfrac{3a - 5}{2a - 3}$

33. $\dfrac{(x - 3)(x + 1)}{1} \cdot \dfrac{(x - 1)(x + 1)}{4(x - 1)}$
$\dfrac{(x^2 - 2x - 3)(x + 1)}{4}$
$\dfrac{x^3 - x^2 - 5x - 3}{4}$

37. $\dfrac{3y}{y^2 + 5y + 6} - \dfrac{5}{-(y^2 - 4)}$
$\dfrac{3y}{y^2 + 5y + 6} + \dfrac{5}{y^2 - 4}$
$\dfrac{3y}{(y + 3)(y + 2)} + \dfrac{5}{(y - 2)(y + 2)}$
$\dfrac{3y(y - 2)}{(y + 3)(y + 2)(y - 2)} + \dfrac{5(y + 3)}{(y - 2)(y + 2)(y + 3)}$
$\dfrac{3y^2 - 6y + 5y + 15}{(y + 3)(y + 2)(y - 2)}$
$\dfrac{3y^2 - y + 15}{y^3 + 3y^2 - 4y - 12}$

41. $\dfrac{(x-5)(x+5)}{2(x+5)} \cdot \dfrac{1}{(x-5)(x-5)}$

$\dfrac{1}{2x-10}$

45. $\dfrac{ab(1+\frac{1}{a})}{ab(2-\frac{3}{ab})}$

$\dfrac{ab+b}{2ab-3}$

49. $\dfrac{6(\frac{2}{3}-\frac{1}{2})}{6(\frac{2}{3}+\frac{1}{2})}$

$\dfrac{2(2)-3(1)}{2(2)+3(1)}$

$\dfrac{1}{7}$

53. $\dfrac{(x+2)(5-\frac{3}{x+2})}{(x+2)(3+\frac{2}{x+2})}$

$\dfrac{5(x+2)-3}{3(x+2)+2}$

$\dfrac{5x+7}{3x+8}$

57. $\dfrac{[(a-b)(a+b)](\frac{a}{a-b}+\frac{a}{a+b})}{[(a-b)(a+b)](\frac{3}{(a-b)(a+b)})}$

$\dfrac{a(a+b)+a(a-b)}{3}$

$\dfrac{2a^2}{3}$

61. $\dfrac{2d}{\frac{d}{r_1}+\frac{d}{r_2}} \cdot \dfrac{r_1r_2}{r_1r_2}$

$\dfrac{2dr_1r_2}{dr_2+dr_1}$

$\dfrac{2dr_1r_2}{d(r_1+r_2)}$

$\dfrac{2r_1r_2}{r_1+r_2}$

65. $\dfrac{P}{Q}+\dfrac{R}{S}=\dfrac{PS}{QS}+\dfrac{RQ}{SQ}=\dfrac{PS+QR}{QS}$

$\dfrac{P}{Q}-\dfrac{R}{S}=\dfrac{PS}{QS}-\dfrac{RQ}{SQ}=\dfrac{PS-QR}{QS}$

69. The following is a solution for the TI–81. It also works for even integers:

```
Prgm6:PRIME
:Input N
:2 → D
:If FPart (N/D)=0
:Goto 5
:√ N → L
:3 → D
:Lbl 1
:If FPart (N/D)=0
:Goto 5
:D+2 → D
:If D ≤ L
:Goto 1
:Disp "PRIME":Stop
:Lbl 5
:Disp D
:N/D → N
:Disp N
```

Exercise 1–5

1. $\sqrt{17^2}=17$

5. $\sqrt[3]{(-5)^3}=-5$

9. $\sqrt{25}\sqrt{x^6}\sqrt{y^8}$

$5\,|x^3|\,|y^4|$

$5y^4\,|x^3|$

13. $\sqrt{(x^2-3)^2}$

$|\,x^2-3\,|$

17. $\sqrt{2^3\cdot 5^2}$

$\sqrt{2^2\cdot 5^2}\,\sqrt{2}$

$2\cdot 5\sqrt{2}$

$10\sqrt{2}$

21. $\sqrt{2^3\cdot 5^2x^2y^9z^{12}}$

Break the radical into two parts
- one with even exponents
- the other what's left over

$\sqrt{2^2 5^2 x^2 y^8 z^{12}}\,\sqrt{2y}$

$10xy^4z^6\sqrt{2y}$

25. $\sqrt[3]{a^4}\,\sqrt[3]{a}\,\sqrt[3]{a^7}$

$\sqrt[3]{a^{12}}$

a^4

29. $\dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$\dfrac{8\sqrt{2}}{2}$

$4\sqrt{2}$

33. $\dfrac{\sqrt{8}}{\sqrt{27}}$

$\dfrac{2\sqrt{2}}{3\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{2\sqrt{6}}{9}$

37. $\dfrac{\sqrt{12x^5y^6}}{\sqrt{5z}} \cdot \dfrac{\sqrt{5z}}{\sqrt{5z}}$

$\dfrac{\sqrt{2^2\cdot 3\cdot 5x^5y^6z}}{5z}$

$\dfrac{2x^2y^3\sqrt{15xz}}{5z}$

41. $\sqrt[3]{\dfrac{16a^5}{b^7c^2} \cdot \dfrac{b^2c}{b^2c}}$

Want exponents in denominator divisible by 3

$\dfrac{\sqrt[3]{2^4a^5b^2c}}{\sqrt[3]{b^9c^3}}$ Simplify all terms in numerator with exponents ≥ 3

$\dfrac{2a\sqrt[3]{2a^2b^2c}}{b^3c}$

45. $\sqrt[4]{\dfrac{8x^5y^7}{50x^3y^9}}$

$\sqrt[4]{\dfrac{4x^2}{25y^2}}$

$\sqrt[4]{\dfrac{4x^2}{5^2y^2} \cdot \dfrac{5^2y^2}{5^2y^2}}$

$\dfrac{\sqrt[4]{100x^2y^2}}{\sqrt[4]{5^4y^4}}$

$\dfrac{\sqrt[4]{10^2x^2y^2}}{5y}$

$\dfrac{\sqrt{10xy}}{5y}$

49. $5\sqrt{3}+7\sqrt{2^2\cdot 3}-\sqrt{3\cdot 5^2}$

$5\sqrt{3}+14\sqrt{3}-5\sqrt{3}$

$14\sqrt{3}$

53. $\sqrt[3]{2^4}+5\sqrt[3]{2^3\cdot 3}$

$2\sqrt[3]{2}+10\sqrt[3]{3}$

57. $2(3a)-4\sqrt{3^4a^2}$

$6a-36a$

$-30a$

61. $(3-4\sqrt{x})(4-2\sqrt{x})$

$12-6\sqrt{x}-16\sqrt{x}+8x$

$12-22\sqrt{x}+8x$

65. $(2\sqrt{2x^2}-\sqrt{4x})(2\sqrt{2x^2}-\sqrt{4x})$

$4(2x^2)-2\sqrt{8x^3}-2\sqrt{8x^3}+4x$

$8x^2-4\sqrt{8x^3}+4x$

$8x^2-8x\sqrt{2x}+4x$

69. $\dfrac{a+\sqrt{b}}{a-\sqrt{b}} \cdot \dfrac{a+\sqrt{b}}{a+\sqrt{b}}$

$\dfrac{a^2+a\sqrt{b}+a\sqrt{b}+b}{a^2+a\sqrt{b}-a\sqrt{b}-b}$

$\dfrac{a^2+2a\sqrt{b}+b}{a^2-b}$

73. $\dfrac{1}{3}\cdot\dfrac{\sqrt{7}}{5}-\dfrac{\sqrt{2}}{3}\cdot\dfrac{\sqrt{2}}{5}$

$\dfrac{\sqrt{7}}{15}-\dfrac{4}{15}$

$\dfrac{\sqrt{7}-4}{15}$

77. $\sqrt{\dfrac{3-\frac{1}{\sqrt{2}}}{2}}$

$\sqrt{\frac{1}{2}\left(3-\frac{\sqrt{2}}{2}\right)}$

$\sqrt{\frac{1}{2}\cdot\frac{6-\sqrt{2}}{2}}$

$\sqrt{\dfrac{6-\sqrt{2}}{4}}$

$\dfrac{\sqrt{6-\sqrt{2}}}{2}$

81. $\sqrt[4]{a^6}$

$\sqrt[4\div 2]{a^{6\div 2}}$

$\sqrt[2]{a^3}$

$\sqrt{a^3}$

$a\sqrt{a}$

85. Recall, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
and $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.

(a) Using $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$,

let $a = \sqrt[3]{x}$ and $b = \sqrt[3]{y}$ so that

$$x - y = (\sqrt[3]{x} - \sqrt[3]{y})((\sqrt[3]{x})^2 + \sqrt[3]{x}\,\sqrt[3]{y} + (\sqrt[3]{y})^2)$$
$$= (\sqrt[3]{x} - \sqrt[3]{y})(\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2})$$

Thus, $Q(x,y) = \sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2}$.

(b) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$; let $a = \sqrt[3]{x}$ and $b = \sqrt[3]{y}$:

$$x + y = (\sqrt[3]{x} + \sqrt[3]{y})(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2}).$$

(c) $8x - y = (2\sqrt[3]{x})^3 - (\sqrt[3]{y})^3$

$$= (2\sqrt[3]{x} - \sqrt[3]{y})(\,(2\sqrt[3]{x})^2 + 2\sqrt[3]{x}\,\sqrt[3]{y} + (\sqrt[3]{y})^2)$$
$$= (2\sqrt[3]{x} - \sqrt[3]{y})(\,4\sqrt[3]{x^2} + 2\sqrt[3]{xy} + \sqrt[3]{y^2})$$

$$8x - y = (\sqrt{8x} - \sqrt{y})(\sqrt{8x} + \sqrt{y})$$
$$= (2\sqrt{2x} - \sqrt{y})(2\sqrt{2x} + \sqrt{y})$$

89. $\sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}} \cdot \dfrac{2-\sqrt{3}}{2-\sqrt{3}}}$

$\sqrt{\dfrac{(2-\sqrt{3})^2}{1}}$

$\sqrt{(2-\sqrt{3})^2}$

$2 - \sqrt{3}$

$\dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} \cdot \dfrac{\sqrt{3} - 1}{\sqrt{3} - 1}$

$\dfrac{4 - 2\sqrt{3}}{2}$

$2 - \sqrt{3}$

Exercise 1–6

1. $64^{\frac{1}{4}} = \sqrt[4]{64} = \sqrt[4]{2^6} = 2\sqrt[4]{2^2} = 2\sqrt{2}$

5. $(-8)^{-\frac{2}{3}} = \dfrac{1}{(-8)^{\frac{2}{3}}} = \dfrac{1}{(\sqrt[3]{-8})^2} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$

9. $(81x^3)^{\frac{1}{4}} = \sqrt[4]{3^4x^3} = 3\sqrt[4]{x^3}$

13. $(8x^3)^{\frac{1}{2}} = \sqrt{8x^3} = 2x\sqrt{2x}$

17. $2a^{\frac{1}{2}}(4a^{\frac{1}{4}}) = 8a^{\frac{1}{2}+\frac{1}{4}} = 8a^{\frac{3}{4}}$

Remember that all rules for exponents apply to rational exponents also.

21. $(4x^{\frac{4}{7}}y^{\frac{6}{5}}z)^{\frac{1}{2}} = 4^{\frac{1}{2}}\,x^{\frac{4}{7}\cdot\frac{1}{2}}\,y^{\frac{6}{5}\cdot\frac{1}{2}}\,z^{\frac{1}{2}} = 2x^{\frac{2}{7}}\,y^{\frac{3}{5}}\,z^{\frac{1}{2}}$

25. $\dfrac{ab^{\frac{1}{3}}}{a^2b^{\frac{5}{6}}} = \dfrac{1}{a^{2-1}b^{\frac{5}{6}-\frac{1}{3}}} = \dfrac{1}{ab^{\frac{1}{2}}}$

29. $\dfrac{6a^{\frac{1}{2}}}{2a^{\frac{1}{4}}} = 3a^{\frac{1}{2}-\frac{1}{4}} = 3a^{\frac{1}{4}}$

33. $\left(\dfrac{x^{\frac{4}{3}}y^{\frac{8}{5}}}{z^{-8}}\right)^{\frac{1}{8}} = \left(x^{\frac{4}{3}}y^{\frac{8}{5}}z^8\right)^{\frac{1}{8}}$
$= x^{\frac{4}{3}\cdot\frac{1}{8}}y^{\frac{8}{5}\cdot\frac{1}{8}}z^{8\cdot\frac{1}{8}} = x^{\frac{1}{6}}\,y^{\frac{1}{5}}\,z$

37. $\left(\dfrac{2^{\frac{m+n}{m}}x^{\frac{n}{m}}}{2^{\frac{n}{m}}x^{\frac{n}{m}}y^{\frac{-n}{m}}}\right)^m$
$= \dfrac{2^{m+n}x^n}{2^nx^ny^{-n}}$
$= 2^my^n$

41. -2.8854 200 $\boxed{x^{1/y}}$ 5 $\boxed{=}$ $\boxed{+/-}$ $\boxed{\texttt{- 2.885399812}}$

$\boxed{\text{TI-81}}$: $\boxed{(}$ $\boxed{(-)}$ 200 $\boxed{)}$ $\boxed{\wedge}$ 5 $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

45. 15.1539 37.5 $\boxed{x^y}$ 3 $\boxed{=}$ $\boxed{x^{1/y}}$ 4 $\boxed{=}$ $\boxed{\texttt{15.15386629}}$

$\boxed{\text{TI-81}}$: $\boxed{(}$ 37.5 $\boxed{\text{MATH}}$ 3 $\boxed{)}$ $\boxed{\wedge}$ 4 $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

49. 15.8322 500 $\boxed{x^{1/y}}$ 3 $\boxed{=}$ $\boxed{y^x}$ $\boxed{(}$ 4 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{=}$

$\boxed{\texttt{15.83223488}}$

$\boxed{\text{TI-81}}$: $\boxed{(}$ $\boxed{\text{MATH}}$ 4 $\boxed{(-)}$ 500 $\boxed{)}$ $\boxed{\wedge}$ $\boxed{(}$
4 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\text{ENTER}}$

53. 849.1202 18 $\boxed{x^y}$ 7 $\boxed{=}$ $\boxed{x^{1/y}}$ 3 $\boxed{=}$ $\boxed{\texttt{849.1202117}}$

$\boxed{\text{TI-81}}$: $\boxed{\text{MATH}}$ 4 $\boxed{(}$ 18 $\boxed{\wedge}$ 7 $\boxed{)}$ $\boxed{\text{ENTER}}$

57. To find the new value of D_1 we replace L_b by $4L_b$.
$D_1 = c(4L_b)^{1.5} = c(4^{1.5})L_b{}^{1.5}$.
Compare this new value to the original value of $cL_b{}^{1.5}$:

$$\frac{c(4^{1.5})L_b{}^{1.5}}{cL_b{}^{1.5}} = 4^{1.5} = 4^{3/2} = (\sqrt{4})^3 = 8$$

Thus *the new leg diameter must be 8 times the original diameter* if the body length increases by a factor of 4.

61. $\dfrac{1}{\sqrt{5}}\left(\left(\dfrac{1+\sqrt{5}}{2}\right)^2 - \left(\dfrac{1-\sqrt{5}}{2}\right)^2\right) = \dfrac{1}{\sqrt{5}}\left(\dfrac{(1+\sqrt{5})^2}{4} - \dfrac{(1-\sqrt{5})^2}{4}\right)$

$= \dfrac{1}{\sqrt{5}}\left(\dfrac{6+2\sqrt{5}}{4} - \dfrac{6-2\sqrt{5}}{4}\right) = \dfrac{1}{\sqrt{5}}\left(\dfrac{(6+2\sqrt{5})-(6-2\sqrt{5})}{4}\right)$

$= \dfrac{1}{\sqrt{5}}\left(\dfrac{4\sqrt{5}}{4}\right) = \dfrac{4}{4} = 1$

Exercise 1–7

1. $-3 - 2 + 5i + 3i$
$-5 + 8i$

5. $-16 + 56i + 6i - 21i^2$
$-16 + 21 + 56i + 6i$
$5 + 62i$

9. $(5 - 2i)^2$
$(5 - 2i)(5 - 2i)$
$25 - 10i - 10i + 4i^2$
$25 - 4 - 20i$
$21 - 20i$

13. $\dfrac{3-4i}{2+5i} \cdot \dfrac{2-5i}{2-5i}$
$\dfrac{6 - 15i - 8i + 20i^2}{4 - 10i + 10i - 25i^2}$
$\dfrac{-14 - 23i}{29}$
$-\dfrac{14}{29} - \dfrac{23}{29}i$

17. $\dfrac{6i}{2-3i} \cdot \dfrac{2+3i}{2+3i}$
$\dfrac{12i + 18i^2}{4 + 6i - 6i - 9i^2}$
$\dfrac{-18 + 12i}{13}$
$-\dfrac{18}{13} + \dfrac{12}{13}i$

21. $\sqrt{5}\,\sqrt{-10}$
$\sqrt{5}\,\sqrt{10}\,i$
$\sqrt{50}\,i$
$5\sqrt{2}\,i$

25. $(3 - \sqrt{-3})(4 + \sqrt{-3})$
$(3 - \sqrt{3}\,i)(4 + \sqrt{3}\,i)$
$12 + 3\sqrt{3}\,i - 4\sqrt{3}\,i - 3i^2$
$12 + 3 - \sqrt{3}\,i$
$15 - \sqrt{3}\,i$

29. $\frac{4-\sqrt{-6}}{2+3\sqrt{-2}}$

$\frac{4-\sqrt{6}\,i}{2+3\sqrt{2}\,i}\cdot\frac{2-3\sqrt{2}\,i}{2-3\sqrt{2}\,i}$

$\frac{8-12\sqrt{2}\,i-2\sqrt{6}\,i+3\sqrt{12}\,i^2}{4-6\sqrt{2}\,i+6\sqrt{2}\,i-9(2)i^2}$

$\frac{8-3\sqrt{12}-12\sqrt{2}i-2\sqrt{6}\,i}{4+18}$

$\frac{8-6\sqrt{3}-12\sqrt{2}i-2\sqrt{6}\,i}{22}$

$\frac{2[(4-3\sqrt{3})-(6\sqrt{2}-\sqrt{6})]i}{22}$

$\frac{4-3\sqrt{3}}{11}-\frac{6\sqrt{2}-\sqrt{6}}{11}i$

33. $\frac{\sqrt{-6}+\sqrt{6}}{\sqrt{-2}-\sqrt{2}}$

$\frac{\sqrt{6}\,i+\sqrt{6}}{\sqrt{2}\,i-\sqrt{2}}=\frac{\sqrt{6}+\sqrt{6}\,i}{-\sqrt{2}+\sqrt{2}\,i}$

$\frac{\sqrt{6}+\sqrt{6}\,i}{-\sqrt{2}+\sqrt{2}\,i}\cdot\frac{-\sqrt{2}-\sqrt{2}\,i}{-\sqrt{2}-\sqrt{2}\,i}$

$\frac{-\sqrt{12}-\sqrt{12}\,i-\sqrt{12}\,i-\sqrt{12}\,i^2}{2+2\sqrt{2}\,i-2\sqrt{2}\,i-2i^2}$

$\frac{-2\sqrt{12}\,i}{4}=\frac{-4\sqrt{3}\,i}{4}=-\sqrt{3}\,i$

37. $i^{15}=i^{12}\cdot i^3=1\cdot i^3=i^2\cdot i=-i$

41. $i^{-15}=\frac{1}{i^{15}}=\frac{1}{i^3}=\frac{1}{-i}=\frac{1}{-i}\cdot\frac{i}{i}=\frac{i}{1}=i$

45. $\frac{(2-i)(9+2i)}{(2-i)-(9+2i)}$

$\frac{18+4i-9i-2i^2}{-7-3i}$

$\frac{20-5i}{-7-3i}$

$\frac{-20+5i}{7+3i}\cdot\frac{7-3i}{7-3i}$

$\frac{-140+60i+35i-15i^2}{49-21i+21i-9i^2}$

$\frac{-125+95i}{58}=-\frac{125}{58}+\frac{95}{58}i$

49. $T\;=\frac{(10-3i)+(20+i)}{(10-3i)(20+i)}$

$=\frac{30-2i}{203-50i}\cdot\frac{203+50i}{203+50i}$

$=\frac{6090+1500i-406i-100i^2}{41209+10150i-10150i-2500i^2}$

$=\frac{6190+1094i}{43709}$

$=\frac{6190}{43709}+\frac{1094}{43709}i$

53. As above, $x-16<0$, so $x<16$.

57. After approximately 18 iterations the value of z repeats the value $0.1074991191+0.0636941246i$.
The following is a program for a TI–81:

```
Prgm2: JULIA
:.5→A
:–.2→B
:Lbl 1
:A²–B²→C
:2AB→D
:C+.1→A
:D+.05→B
:Disp A
:Disp B
:Pause
:Goto 1
```

Chapter 1 Review

1. $\{0,1,2,3,4,5,6,7\}$

3. $\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\ldots,\frac{100}{101}\right\}$

5. $0.230769\overline{230769}$

7. $\frac{5}{6}-\frac{3}{8}-\frac{3}{4}$

$\frac{20}{24}-\frac{9}{24}-\frac{18}{24}$

$-\frac{7}{24}$

9. $\frac{5(5-2(9-7)+\frac{4}{5})-(8-12)(2-8)}{5(3-7)^3-(3-7)^2}$

$\frac{5(\frac{9}{5})-24}{-336}=\frac{-15}{-336}=\frac{5}{112}$

11. $\frac{b(a+3)+4a(3b+2)}{4a(b)}$

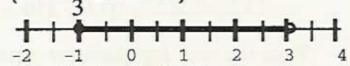
$\frac{8a+13ab+3b}{4ab}$

13. $\frac{6a^2+b^2}{3ab}\cdot\frac{3b}{5a}$

$\frac{6a^2+b^2}{5a^2}$

15. $(-\frac{1}{4},\infty)$

17. $\{\,x\mid -\frac{\pi}{3}\le x<\pi\,\}$



19. $[\pi,\infty)$; $\{\,x\mid x\ge\pi\,\}$

21. $-(-\pi-9)$

$\pi+9$

23. $5x^2$

25. $5|\,2x-1\,|$

$27.27x^6y^{-3}$

$\frac{27x^6}{y^3}$

29. $\frac{x^3y^2}{25}$

31. $-\frac{36x^6}{y^6}$

33. $\left(\frac{-2x^{-2}y^{-1}}{8x^{-2}y^{-3}}\right)^3$

$\left(\frac{-x^{-2}y^{-1}}{4x^{-2}y^{-3}}\right)^3$

$\frac{-x^{-6}y^{-3}}{64x^{-6}y^{-9}}$

$-\frac{y^6}{64}$

35. $\frac{1}{x^{2n}}$

37. 4.2182×10^{16}

39. 0.000 000 405 2

41. $2^3-2(2^2)+12(2)+3$

27

43. $-2(-6)(\frac{1}{3}(4(\frac{1}{2}(-6)(-(-6)+4)-2)-7)+2)+1$

$-2(-6)(\frac{1}{3}(4(\frac{1}{2}(-6)(10)-2)-7)+2)+1$

$-2(-6)(\frac{1}{3}(4(-30-2)-7)+2)+1$

$-2(-6)(\frac{1}{3}(4(-32)-7)+2)+1$

$-2(-6)(\frac{1}{3}(-128-7)+2)+1$

$-2(-6)(\frac{1}{3}(-135)+2)+1$

$-2(-6)(-45+2)+1$

$-2(-6)(-43)+1$

-515

45. $(a^2-10a+25)(5a+1)$

$5a^3-49a^2+115a+25$

47. $-3x^4-x^2+1$

49. $y^3 + y^2 + y + 1$

$$\begin{array}{r}
y^3 + y^2 + y + 1 \\
y-1 \overline{\big) y^4 + 0y^3 + 0y^2 + 0y - 1} \\
\underline{y^4 - y^3 \qquad\qquad\qquad\quad} \\
y^3 + 0y^2 + 0y - 1 \\
\underline{y^3 - y^2 \qquad\qquad\quad} \\
y^2 + 0y - 1 \\
\underline{y^2 - y \qquad} \\
y - 1 \\
\underline{y - 1}
\end{array}$$

51. $x^3(25 - x^2)$
$x^3(5 - x)(5 + x)$

53. $a(8a^2 - 14a + 5)$
$a(2a - 1)(4a - 5)$

55. $b(8a^3 + 125b^3)$
$b(2a + 5b)(4a^2 - 10ab + 25b^2)$

57. $(5x - y)(x - 10y)$

59. $2(27x^6 - y^3)$
$2(3x^2 - y)(9x^4 + 3x^2y + y^2)$

61. $(x + 2y)(3a - b)$

63. $7a^2 - 32a - 21$

65. $3(x^3 - 1)^2 - 2(x^3 - 1) - 8$
$u = x^3 - 1$
$3u^2 - 2u - 8$
$(3u + 4)(u - 2)$
$[3(x^3 - 1) + 4][(x^3 - 1) - 2]$
$(3x^3 + 1)(x^3 - 3)$

67. $[a - 2(x + 5y)][a + 2(x + 5y)]$
$(a - 2x - 10y)(a + 2x + 10y)$

69. $3x^2(x^3y^9 + 27z^6)$
$3x^2(xy^3 + 3z^2)(x^2y^6 - 3xy^3z^2 + 9z^4)$

71. $\dfrac{a(3a - 1)}{(3a - 1)(3a + 1)}$

$\dfrac{a}{3a + 1}$

73. $\dfrac{(2x - 1)(2x + 1)}{(2x - 1)(4x^2 + 2x + 1)}$

$\dfrac{2x + 1}{4x^2 + 2x + 1}$

75. $\dfrac{3y(x - 2y) + 5x(3x - y)}{15xy}$

$\dfrac{15x^2 - 2xy - 6y^2}{15xy}$

77. $\dfrac{1}{4x} - \dfrac{2x}{4x - 5}$

$\dfrac{4x - 5 - 8x^2}{4x(4x - 5)}$

$\dfrac{-8x^2 + 4x - 5}{4x(4x - 5)}$

79. $\dfrac{2x(x - 1)^2}{(x + 4)(x - 1)} \bullet \dfrac{(x - 4)(x + 4)}{(x - 1)(4x + 3)}$

$\dfrac{2x(x - 4)}{4x + 3}$

81. $\dfrac{3x}{(x - 3)(x - 2)} - \dfrac{2x - 5}{x - 3}$

$\dfrac{3x}{(x - 3)(x - 2)} - \dfrac{2x - 5}{x - 3} \bullet \dfrac{x - 2}{x - 2}$

$\dfrac{3x - (2x - 5)(x - 2)}{(x - 3)(x - 2)}$

$\dfrac{-2x^2 + 12x - 10}{(x - 3)(x - 2)}$

83. $\dfrac{\left(\dfrac{5}{2x} - \dfrac{3}{y}\right) \bullet 2xy}{\left(\dfrac{1}{x} + \dfrac{2}{y}\right) \bullet 2xy}$

$\dfrac{5y - 6x}{2(y + 2x)}$

85. $\dfrac{\left(\dfrac{3}{a - b} - 2\right)(a - b)}{\left(5 - \dfrac{3}{a - b}\right)(a - b)}$

$\dfrac{3 - 2a + 2b}{5a - 5b - 3}$

87. 4

89. $\sqrt{2 \bullet 3^3x^4y^7z^2} = \sqrt{3^2x^4y^6z^2} \bullet \sqrt{2 \bullet 3y}$
$= 3x^2y^3z\sqrt{6y}$

91. $\left(\sqrt[3]{9a^4b^2}\right)^2 = (a\sqrt[3]{3^2ab^2})^2 = a^2\sqrt[3]{3^4a^2b^4} = 3a^2b\sqrt[3]{3a^2b}$

93. $b\sqrt[6]{a^4} = b\sqrt[3]{a^2}$

95. $\dfrac{18a}{\sqrt{2^2 \bullet 3^3a^3}} = \dfrac{3}{\sqrt{3a}} \bullet \dfrac{\sqrt{3a}}{\sqrt{3a}} = \dfrac{\sqrt{3a}}{a}$

97. $-6ab\sqrt{2b} + 15ab\sqrt{2b}$
$= 9ab\sqrt{2b}$

99. $3xy^2 - 3xy\sqrt{y} + 3y^2\sqrt{2x}$

101. $\dfrac{\sqrt{6x} - 5}{\sqrt{6x} + \sqrt{2}} \bullet \dfrac{\sqrt{6x} - \sqrt{2}}{\sqrt{6x} - \sqrt{2}}$

$\dfrac{6x - 5\sqrt{6x} - 2\sqrt{3x} + 5\sqrt{2}}{2(3x - 1)}$

103. $\dfrac{3}{8\sqrt{2}} + \dfrac{2\sqrt{3}}{8\sqrt{2}}$

$\dfrac{(3 + 2\sqrt{3}) \bullet \sqrt{2}}{8\sqrt{2} \bullet \sqrt{2}}$

$\dfrac{3\sqrt{2} + 2\sqrt{6}}{16}$

105. $25^{\frac{1}{2}}(x^2)^{\frac{1}{2}}x^{\frac{1}{2}} = 5x\sqrt{x}$

107. $\dfrac{1}{3x\sqrt{x}} = \dfrac{\sqrt{x}}{3x^2}$

109. $\dfrac{\sqrt{2}|x^3|}{4|y^5|}$

111. $(x^{\frac{17}{12}}y^{\frac{1}{4}})$

113. $\dfrac{4^{-\frac{1}{2}}x^{\frac{3}{4}}y^{-\frac{1}{2}}}{x^{\frac{1}{2}}y^{\frac{3}{4}}}$

$\dfrac{x^{\frac{1}{4}}}{2y^{\frac{5}{4}}}$

115. $x^{\frac{c}{2}}y^{\frac{b}{3}}$

117. 50.2304

119. 3.5342

121. $-19\frac{1}{2} + 15i$

123. $-2 + \frac{2}{9}i + 9i - i^2 - (\frac{1}{9} - 2i + 9i^2)$

$-2 + 9\frac{2}{9}i + 1 - \frac{1}{9} + 2i + 9$

$7\frac{8}{9} + 11\frac{2}{9}i$

125. $\dfrac{-3 + 2i}{1 + 4i} \bullet \dfrac{1 - 4i}{1 - 4i} = \frac{5}{17} + \frac{14}{17}i$

127. $\dfrac{6 + 6\sqrt{3}i}{4 - 2\sqrt{3}i} = \dfrac{3 + 3\sqrt{3}i)}{2 - \sqrt{3}i} \bullet \dfrac{2 + \sqrt{3}i}{2 + \sqrt{3}i} =$

$\dfrac{-3 + 9\sqrt{3}i}{7} = -\frac{3}{7} + \frac{9}{7}\sqrt{3}i$

129. $\dfrac{-3 + 7i}{24 + 2i} = -\frac{1}{10} + \frac{3}{10}i$

131. $\dfrac{1}{t} = \dfrac{5 - 2i}{7 - 11i}$

$t = \dfrac{7 - 11i}{5 - 2i} = \frac{57}{29} - \frac{41}{29}i$

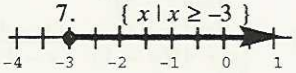
Chapter 1 Test

1. $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}$

3. $\frac{5}{4} - \frac{3}{8} - \frac{2}{3}$
$\frac{30}{24} - \frac{9}{24} - \frac{16}{24}$
$\frac{5}{24}$

5. $\left(\frac{2a}{b} + \frac{b}{3a}\right) \cdot \left(\frac{3b}{5a}\right)$
$\frac{6a^2 + b^2}{3ab} \cdot \frac{3b}{5a}$
$\frac{6a^2 + b^2}{5a^2}$

7. $\{ x \mid x \geq -3 \}$



9. $-\mid -4 \mid$
$-(4)$
-4

11. $\mid -3x^2y \mid$
$|-3|\ |x^2|\ |y|$
$3x^2\ |y|$

13. $(-2^2x^{-1}y^5)(2^2x^5y^{-1}z^0)$
$(-4)(4)(x^4y^4)(1)$
$-16x^4y^4$

15. $\left(\frac{2a^{-2}}{12a^{-12}}\right)^{-2}$
$\left(\frac{1}{6}a^{10}\right)^{-2}$
$(6^{-1}a^{10})^{-2}$
6^2a^{-20}
$\frac{36}{a^{20}}$

17. 2.05×10^{11}

19. $(9a^2 - 6a + 1)(a + 1)$
$\left(9 \cdot \frac{4}{9} - 6(-\frac{2}{3}) + 1\right)\left(-\frac{2}{3} + 1\right)$
$(4 + 4 + 1)(\frac{1}{3})$
3

21. $-2x^3(x^3 + \frac{1}{2}x - 3 - 2x^{-3})$
$-2x^6 - x^4 + 6x^3 + 4x^0$
$-2x^6 - x^4 + 6x^3 + 4$

23. $\frac{2x^3 - x^2 + 4x - 5}{x - 2} = 2x^2 + 3x + 10 + \frac{15}{x - 2}$

$$
\begin{array}{r|rrrr}
 & 2x^2 & +\ 3x & +\ 10 & \\
\cline{2-5}
x - 2 & 2x^3 & -\ x^2 & +\ 4x & -\ 5 \\
 & 2x^3 & -\ 4x^2 & & \\
\cline{2-5}
 & & 3x^2 & +\ 4x & \\
 & & 3x^2 & -\ 6x & \\
\cline{2-5}
 & & & 10x & -\ 5 \\
 & & & 10x & -\ 20 \\
\cline{4-5}
 & & & & 15
\end{array}
$$

25. $9x^2 - 3x - 2$
$(3x - 2)(3x + 1)$

27. $64x^6 - 1$
$2^6x^6 - 1$
$(2x)^6 - 1$
$[(2x)^3 - 1][(2x)^3 + 1]$
$[(2x - 1)(4x^2 + 2x + 1)]$
$[(2x + 1)(4x^2 - 2x + 1)]$

29. $3ac - 2bd + ad - 6bc$
$3ac + ad - 6bc - 2bd$
$a(3c + d) - 2b(3c + d)$
$(3c + d)(a - 2b)$

31. $\frac{2x^2 + x - 1}{4x^2 - 1}$
$\frac{(2x - 1)(x + 1)}{(2x - 1)(2x + 1)}$
$\frac{x + 1}{2x + 1}$

33. $\frac{x}{2x + 1} - \frac{x - 1}{3x}$
$\frac{3x(x) - (x - 1)(2x + 1)}{(2x + 1)(3x)}$
$\frac{3x^2 - (2x^2 - x - 1)}{3x(2x + 1)}$
$\frac{x^2 + x + 1}{3x(2x + 1)}$

35. $\frac{\frac{2}{3a} - \frac{3}{b}}{\frac{2}{3ab} + 2} \cdot \frac{3ab}{3ab}$
$\frac{2(b) - 3(3a)}{2 + 2(3ab)}$
$\frac{2b - 9a}{2 + 6ab}$

37. $\sqrt[3]{128}$
$\sqrt[3]{2^7}$
$\sqrt[3]{2^6}\sqrt[3]{2}$
$2^2\sqrt[3]{2}$
$4\sqrt[3]{2}$

39. $\left(\sqrt[3]{16ab^5}\right)^2$
$\left(\sqrt[3]{2^4\, ab^5}\right)^2$
$\left(2b\ \sqrt[3]{2ab^2}\right)^2$
$(2b)^2\left(\sqrt[3]{2ab^2}\right)^2$
$4b^2\ \sqrt[3]{4a^2b^4}$
$4b^2 \cdot b\sqrt[3]{4a^2b}$
$4b^3\ \sqrt[3]{4a^2b}$

41. $\frac{12x^3}{\sqrt{24x^5}}$
$\frac{12x^3}{2x^2\sqrt{6x}} \cdot \frac{\sqrt{6x}}{\sqrt{6x}}$
$\frac{6x\sqrt{6x}}{6x}$
$\sqrt{6x}$

43. $\sqrt{45a^3b} - a\sqrt{20ab} + \sqrt{5a^3b}$
$3a\sqrt{5ab} - 2a\sqrt{5ab} + a\sqrt{5ab}$
$2a\sqrt{5ab}$

45. $\frac{3\sqrt{a} - 3}{\sqrt{6a} + \sqrt{3}} \cdot \frac{\sqrt{6a} - \sqrt{3}}{\sqrt{6a} - \sqrt{3}}$
$\frac{3\sqrt{6a^2} - 3\sqrt{3a} - 3\sqrt{6a} + 3\sqrt{3}}{(6a) - 3}$
$\frac{3a\sqrt{6} - 3\sqrt{3a} - 3\sqrt{6a} + 3\sqrt{3}}{6a - 3}$
$\frac{3(a\sqrt{6} - \sqrt{3a} - \sqrt{6a} + \sqrt{3})}{3(2a - 1)}$
$\frac{a\sqrt{6} - \sqrt{3a} - \sqrt{6a} + \sqrt{3}}{2a - 1}$

47. $\sqrt[3]{2^4x^3}$
$2x\ \sqrt[3]{2}$

49. $\sqrt{20x^6y^8}$
$2\sqrt{5}\ |x^3| \cdot |y^4|$
$2\sqrt{5}\ |x^3|\ y^4$

51. $(x^{\frac{1}{4}}y^{-\frac{3}{4}})(x^{\frac{3}{4}}y^{\frac{1}{2}})$
$x^1\ y^{-\frac{1}{4}}$
$\frac{x}{y^{\frac{1}{4}}}$

53. $\frac{(8x^{\frac{3}{2}}y)^{-\frac{1}{3}}}{2x^{-\frac{1}{3}}y^{\frac{2}{3}}}$
$\frac{8^{-\frac{1}{3}}x^{-\frac{1}{2}}y^{-\frac{1}{3}}}{2x^{-\frac{1}{3}}y^{\frac{2}{3}}})$
$\frac{1}{8^{\frac{1}{3}} \cdot 2 \cdot x^{\frac{1}{2}}x^{-\frac{1}{3}}y^{\frac{1}{3}}y^{\frac{2}{3}}}$
$\frac{1}{4x^{\frac{1}{6}}y}$

55. $(a^{\frac{m}{n}}b^{\frac{2}{n}})^{\frac{n}{2m}}$
$a^{\frac{1}{2}}b^{\frac{1}{m}}$

57. $(-8 + 3i)^2(2 - 7i)$
$(55 - 48i)(2 - 7i)$
$-226 - 481i$

59. $(\sqrt{-4} - \sqrt{-9})(3 + \sqrt{-12})$
$(2i - 3i)(3 + 2\sqrt{3}i)$
$-i(3 + 2\sqrt{3}i)$
$-3i - 2\sqrt{3}(i^2)$
$2\sqrt{3} - 3i$

61. $\frac{1}{T} = \frac{Z_2 + Z_1}{Z_1Z_2}$
$T = \frac{Z_1Z_2}{Z_2 + Z_1} =$
$\frac{(2 + \mathrm{T}i)(1 + 2i)}{(2 + 4i) + (1 + 2i)} =$
$\frac{2}{3} + \frac{4}{3}i$

Freeload Press
Liberating the Textbook

"Don't mess this up…"

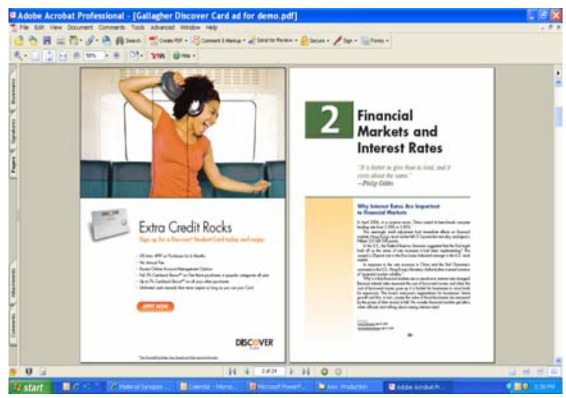
No publisher has used advertising to deliver free textbooks. Until now.
The quote above sums up what customers are telling us about this opportunity.

Students and instructors love our "freeload" model. And they know that advertisers are paying the textbook bill.
They just want us to be smart about how ads are placed in textbooks.

A collaboration of academics and advertisers developed ***StudyBreak Ads™*** --a patent-pending system that places advertising in textbooks in a manner that works for students, instructors and advertisers.

StudyBreak Ads™ feature an academic approval process with guidelines for:

- ✓ Which industries/products
- ✓ Types/size of ads
- ✓ Where ads are placed
- ✓ How frequently ads appear



StudyBreak Ads™ protect the scholarship and the flow of a textbook, assuring that we "don't mess this up."

StudyBreak Ads™

To learn more, write us at info@freeloadpress.com

Chapter 2

Exercise 2–1

1. $13x = 5 - 3x$
   $16x = 5$
   $x = \frac{5}{16}$ $\{\frac{5}{16}\}$

5. $\frac{1}{4}x + 3 = \frac{3}{8}x - 8$
   $8(\frac{1}{4}x + 3) = 8(\frac{3}{8}x - 8)$
   $2x + 24 = 3x - 64$
   $88 = x$ $\{88\}$

9. $-5(3x - 2) + x = 0$
   $-15x + 10 + x = 0$
   $-14x + 10 = 0$
   $10 = 14x$
   $\frac{10}{14} = x$ $\{\frac{5}{7}\}$

13. $6x - 5 = x + 5(x - 1)$
   $6x - 5 = x + 5x - 5$
   $6x - 5 = 6x - 5$
   $0 = 0$ (identity) R

17. $\frac{3}{4}(4x - 3) + x = -17$
   $3(4x - 3) + 4x = -68$
   $12x - 9 + 4x = -68$
   $16x = -59$
   $x = -\frac{59}{16}$ $\{-\frac{59}{16}\}$

21. $-4[2x - 3(3x - 2) - (5 - x)] + 6(1 - x) = 0$
   $-4[2x - 9x + 6 - 5 + x] + 6 - 6x = 0$
   $24x - 4 + 6 - 6x = 0$
   $18x = -2$
   $x = -\frac{1}{9}$ $\{-\frac{1}{9}\}$

25. $5(x - 3) = -(15 - 5x)$
   $5x - 15 = -15 + 5x$
   $0 = 0$ R

29. $150x - 13.8 = 0.04(1500 - 1417x)$
   $150x - 13.8 = 60 - 56.68x$
   $206.68x = 73.8$
   $x = \frac{73.8}{206.68} \approx 0.3571$

33. $2S = 2Vt - gt^2$
   $2S + gt^2 = 2Vt$
   $\frac{2S + gt^2}{2t} = V$

37. $d = d_1 + (k-1)d_2 + (j - d_2)d_3$
   $d = d_1 + kd_2 - d_2 + jd_3 - d_2d_3$
   $d - d_1 - jd_3 = kd_2 - d_2 - d_2d_3$
   $d - d_1 - jd_3 = d_2(k - 1 - d_3)$
   $\frac{d - d_1 - jd_3}{k - 1 - d_3} = d_2$

41. $2x - y = 5x + 6y$; for x
   $-7y = 3x$
   $-\frac{7}{3}y = x$

45. $\frac{x + y}{3} = \frac{2x - y}{5}$; for x
   $5(x + y) = 3(2x - y)$
   $5x + 5y = 6x - 3y$
   $8y = x$

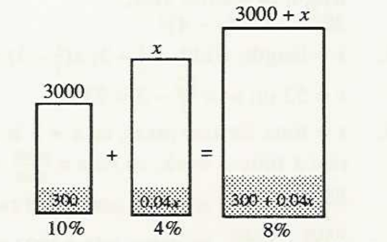
49. $b(y - 4) = a(x + 3)$; for x
   $by - 4b = ax + 3a$
   $by - 4b - 3a = ax$
   $\frac{by - 4b - 3a}{a} = x$

53. $\frac{1}{s} = f + \frac{1 - f}{p}$
   $sp(\frac{1}{s}) = sp(f) + sp(\frac{1 - f}{p})$
   $p = spf + s(1 - f)$
   $p = spf + s - sf$
   $p - s = spf - sf$
   $p - s = sf(p - 1)$
   $\frac{p - s}{s(p - 1)} = f$

57. x = amount at 14% gain;
   $0.14x - 0.09(18000 - x) = 680$
   $0.14x - 1620 + 0.09x = 680$
   $0.23x = 2300$
   $x = \frac{2300}{0.23} = 10000$; thus \$10,000 at 14% gain, \$8,000 at 9% loss.

61. x = amount at 5%;
   $0.05x - 0.09(18000 - x) = 100$
   $0.05x - 1620 + 0.09x = 100$
   $0.14x = 1720$
   $x = \frac{1720}{0.14} = 12285.71$, so \$12,285.71 at 5% and \$5,714.29 at 9%.

65. Let x = amount of 4% pesticide solution.
   The total amount of solution will be $3000 + x$. The total amount of pesticide will be 10% of 3000 (which is 300) and 4% of x, $(0.04x)$, or $300 + 0.04x$.



We want this amount of to be 8% of $3000 + x$:
$0.08(3000 + x) = 300 + 0.04x$
$240 + 0.08x = 300 + 0.04x$
$0.04x = 60$
$x = \frac{60.0}{0.04} = 1500.$
Thus, 1500 gallons of the 4% solution should be added to the 3000 gallons of 10% solution.

69. rate x time = work, so rate = $\frac{\text{work}}{\text{time}}$; first rate is $\frac{50}{3}$ tons/hour; second is $\frac{50}{2\frac{1}{4}} = \frac{200}{9}$ tons/hour. Combined rate is $\frac{50}{3} + \frac{200}{9}$ tons/hour.
   (a) $(\frac{50}{3} + \frac{200}{9})t = 50$; $\frac{350}{9}t = 50$; $t = 50(\frac{9}{350})$; $t = 1\frac{2}{7}$ hour = 1 hr 17 min.
   (b) $(\frac{50}{3} + \frac{200}{9})t = 235$; $\frac{350}{9}t = 235$; $t = 235(\frac{9}{350})$; $t = 6\frac{3}{70}$ hour = 6 hr 3 min.

73. If x is the rate of the truck, that of the car is $x + 15$. $d = rt$, so $t = \frac{d}{r}$, and the times are equal, so $\frac{200}{x + 15} = \frac{150}{x}$. $200x = 150(x + 15)$, $50x = 2250$, $x = 45$ mph for the truck, and 60 mph for the car.

77. x = unknown distance; $t = \frac{d}{r}$, and time for slower – time for faster is 2 minutes = $\frac{2}{60}$ hour, so $\frac{x}{5} - \frac{x}{7} = \frac{2}{60}$, $\frac{2x}{35} = \frac{1}{30}$, $x = \frac{1}{30}(\frac{35}{2}) = \frac{7}{12}$ mile.

Exercise 2–2

1. $x^2 = 7x + 8$
   $x^2 - 7x - 8 = 0$
   $(x - 8)(x + 1) = 0$
   $x - 8 = 0$ or $x + 1 = 0$
   $x = 8$ or $x = -1$ $\{-1, 8\}$

5. $6(\frac{x^2}{6}) = 6(\frac{x}{3}) + 6(\frac{1}{2})$
   $x^2 = 2x + 3$
   $x^2 - 2x - 3 = 0$
   $(x - 3)(x + 1) = 0$ $\{-1, 3\}$

9. $2x(\frac{x}{2}) + 2x(\frac{7}{2}) = 2x(\frac{4}{x})$
   $x^2 + 7x = 8$
   $x^2 + 7x - 8 = 0$
   $(x + 8)(x - 1) = 0$ $\{-8, 1\}$

13. $5x^2 - 6y^2 = 7xy$
   $5x^2 - 7xy - 6y^2 = 0$
   $(5x + 3y)(x - 2y) = 0$
   $5x + 3y = 0$ or $x - 2y = 0$
   $x = -\frac{3y}{5}$ or $x = 2y$ $\{-\frac{3y}{5}, 2y\}$

17. $3y^2 = 27$
   $y^2 = 9$
   $y = \pm 3$ $\{\pm 3\}$

21. $9x^2 - 40 = 0$
   $9x^2 = 40$
   $x^2 = \frac{40}{9}$
   $x = \pm\sqrt{\frac{40}{9}} = \pm\frac{2\sqrt{10}}{3}$ $\{\pm\frac{2\sqrt{10}}{3}\}$

25. $(x - 3)^2 = 10$
   $x - 3 = \pm\sqrt{10}$
   $x = 3 \pm \sqrt{10}$ $\{3 \pm \sqrt{10}\}$

29. $a(bx + c)^2 = d, a, d > 0$
   $(bx + c)^2 = \frac{d}{a}$
   $bx + c = \pm\sqrt{\frac{d}{a}}$
   $bx = -c \pm \frac{\sqrt{ad}}{a}$
   $x = -\frac{c}{b} \pm \frac{\sqrt{ad}}{ab}$ $\{-\frac{c}{b} \pm \frac{\sqrt{ad}}{ab}\}$

33. $2y^2 + \frac{4y}{3} = 5$
   $6y^2 + 4y - 15 = 0$
   $a = 6, b = 4, c = -15$
   $y = \frac{-4 \pm \sqrt{4^2 - 4(6)(-15)}}{2(6)}$
   $y = \frac{-4 \pm \sqrt{376}}{12} = \frac{-4 \pm 2\sqrt{94}}{12}$
   $y = \frac{2(-2 \pm \sqrt{94})}{12} = \frac{-2 \pm \sqrt{94}}{6}$

37. $5x^2 - 8x - 12$
   $5(x - \frac{4 + 2\sqrt{19}}{5})(x - \frac{4 - 2\sqrt{19}}{5})$

41. x = length of longer side; then $x - 4$ is length of shorter side;
$20^2 = x^2 + (x - 4)^2$
$400 = x^2 + x^2 - 8x + 16$
$2x^2 - 8x - 384 = 0$
$x^2 - 4x - 192 = 0$
$(x - 16)(x + 12) = 0$
$x = 16$ or -12
$x > 0$ so $x = 16$.

45. x = length; width $= \frac{x}{2} - 3$; $x(\frac{x}{2} - 3) = 1196$;
$x = 52$ m; $w = \frac{52}{2} - 3 = 23$ m.

49. x = time for one press, so $x + 3$ is the time for the other press; rate x time = work, so $rate = \frac{\text{work}}{\text{time}}$. One rate is $\frac{10000}{x}$, second is $\frac{10000}{x+3}$; combined rate is $\frac{10000}{x} + \frac{10000}{x+3}$, so using rate x time = work we obtain
$(\frac{10000}{x} + \frac{10000}{x+3})8 = 10000$
$(\frac{1}{x} + \frac{1}{x+3})8 = 1$; $\frac{(x+3)+x}{x(x+3)} \cdot 8 = 1$
$\frac{8(2x+3)}{x^2+3x} = 1$; $16x + 24 = x^2 + 3x$
$0 = x^2 - 13x - 24$; $x = \frac{\sqrt{265} + 13}{2}$, so the rates are $\frac{\sqrt{265} + 13}{2} \approx 14.6$ hours and $\frac{\sqrt{265} + 19}{2} \approx 17.6$ hours.

53. $2x - 3 = 0$
$2x = 3$
$x = \frac{3}{2}$
$\{ x \mid x \neq 1\frac{1}{2} \}$

57. $3m^2 - 6m = 0$
$3m(m - 2) = 0$
$m = 0$ or $m - 2 = 0$
$m = 0$ or $m = 2$
$\{ m \mid m \neq 0, 2 \}$

61. $x^2 + 4 > 0$
R because $x^2 \geq 0$ and $4 > 0$, so their sum is greater than or equal to 4, which is greater than 0.

65. $(x - 3)^2 - 4(x - 3) - 9 = 0$
Let $u = x - 3$; $u^2 - 4u - 9 = 0$;
$u = 2 \pm \sqrt{13}$; $x - 3 = 2 \pm \sqrt{13}$;
$x = 5 \pm \sqrt{13}$ $\{5 \pm \sqrt{13}\}$

69. $x^{2/3} - 10x^{1/3} = -9$
$u = x^{1/3}$, so $u^2 = x^{2/3}$
$u^2 - 10u + 9 = 0$
$u = 9$ or 1
$x^{1/3} = 9$ or $x^{1/3} = 1$
$(x^{1/3}) = 9^3$ or $(x^{1/3}) = 1^3$
$x = 729$ or 1 $\{1, 729\}$

73. $a\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)^2 + b\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) + c = a\left(\frac{b^2 - 2b\sqrt{b^2 - 4ac} + b^2 - 4ac}{4a^2}\right) + \frac{-b^2 + b\sqrt{b^2 - 4ac}}{2a} + c$
$= \frac{b^2 - 2b\sqrt{b^2 - 4ac} + b^2 - 4ac}{4a} + \frac{-2b^2 + 2b\sqrt{b^2 - 4ac}}{4a} + \frac{4ac}{4a} = 0.$
The case for $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ is almost the same.

77. $(a + bi)^2 = c + di$
$a^2 - b^2 + 2abi = c + di$
$a^2 - b^2 = c$, $2ab = d$; $b = \frac{d}{2a}$
$a^2 - (\frac{d}{2a})^2 = c$
$4a^4 - 4a^2c - d^2 = 0$; this is quadratic in a^2, so $a^2 = \frac{-(-4c) \pm \sqrt{(-4c)^2 - 4(4)(-d^2)}}{2(4)}$
$a = \pm\sqrt{\frac{4c \pm \sqrt{16c^2 + 16d^2}}{8}} = \pm\sqrt{\frac{4c \pm 4\sqrt{c^2 + d^2}}{8}} = \pm\sqrt{\frac{c \pm \sqrt{c^2 + d^2}}{2}}$. a must be real, so we require that $c \pm \sqrt{c^2 + d^2} \geq 0$; since $\sqrt{c^2 + d^2} \geq c$ we choose $c + \sqrt{c^2 + d^2}$, and choose $a \geq 0$, obtaining $a = \sqrt{\frac{c + \sqrt{c^2 + d^2}}{2}}$. Using $b = \frac{d}{2a}$ it can be shown that $b = \frac{d}{\sqrt{2}(\sqrt{c + \sqrt{c^2 + d^2}})}$ when c,d not both 0. When c and d are zero, let $b = 0$.

Exercise 2–3

1. $(\sqrt{2x+3})^2$
$2x + 3$

5. $(\sqrt{x} - 5)^2$
$(\sqrt{x} - 5)(\sqrt{x} - 5)$
$x - 5\sqrt{x} - 5\sqrt{x} + 25$
$x - 10\sqrt{x} + 25$

9. $(\sqrt{2x} - 2)^2$
$(\sqrt{2x} - 2)(\sqrt{2x} - 2)$
$2x - 2\sqrt{2x} - 2\sqrt{2x} + 4$
$2x - 4\sqrt{2x} + 4$

13. $(2\sqrt{x-1})^2$
$2^2(\sqrt{x-1})^2$
$4(x - 1)$
$4x - 4$

17. $(1 - \sqrt{1-x})^2$
$1 - 2\sqrt{1-x} + (1 - x)$
$-x + 2 - 2\sqrt{1-x}$

21. $\left(\sqrt[3]{\frac{x}{2}}\right)^3 = 3^3$
$\frac{x}{2} = 27$
$x = 54$ $\{54\}$

25. $(\sqrt[5]{x+3})^5 = (-1)^5$
$x + 3 = -1$
$x = -4$ $\{-4\}$

29. $\sqrt[3]{2x+3} = -5$
$(\sqrt[3]{2x+3})^3 = (-5)^3$
$2x + 3 = -125$
$2x = -128$
$x = -64$ $\{-64\}$

33. $(\sqrt[4]{x^2 - 24x})^4 = (3)^4$
$x^2 - 24x = 81$
$x^2 - 24x - 81 = 0$
$x = -3$ or 27 $\{-3, 27\}$

37. $(\sqrt{x^2 - 5x + 2})^2 = 4^2$
$x^2 - 5x + 2 = 16$
$x^2 - 5x - 14 = 0$
$(x - 7)(x + 2) = 0$
$x = 7$ or -2 $\{-2, 7\}$

41. $(\sqrt{2p+5})^2 = (\sqrt{3p+4})^2$
$2p + 5 = 3p + 4$
$1 = p$ $\{1\}$

45. $(\sqrt{y}\sqrt{y-5})^2 = 6^2$
$y(y - 5) = 36$
$y^2 - 5y - 36 = 0$
$(y - 9)(y + 4) = 0$
$y = 9$ or -4
-4 does not check. $\{9\}$

49. $(\sqrt{x-2})^2 = (x-2)^2$
$x - 2 = x^2 - 4x + 4$
$0 = x^2 - 5x + 6$
$0 = (x - 3)(x - 2)$
$x = 3$ or 2 $\{2, 3\}$

53. $(\sqrt{p+1})^2 = (\sqrt{2p+9}-2)^2$
$p+1 = (\sqrt{2p+9}-2)(\sqrt{2p+9}-2)$
$p+1 = (2p+9) - 4\sqrt{2p+9} + 4$
$4\sqrt{2p+9} = p+12$
$(4\sqrt{2p+9})^2 = (p+12)^2$
$16(2p+9) = p^2 + 24p + 144$
$0 = p^2 - 8p$
$0 = p(p-8)$
$p = 0$ or 8 $\{0, 8\}$

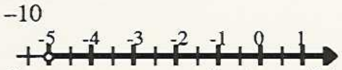
57. $(4x+2)^{1/2} - (2x)^{1/2} = 0$
$[(4x+2)^{1/2}]^2 = [(2x)^{1/2}]^2$
$4x + 2 = 2x$
$2x = -2$
$x = -1$
Does not check. Φ

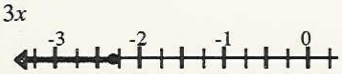
61. $D = \sqrt[3]{\dfrac{6A}{\pi}}$; for A
$D^3 = \left(\sqrt[3]{\dfrac{6A}{\pi}}\right)^3$
$D^3 = \dfrac{6A}{\pi}$
$\pi D^3 = 6A$
$\dfrac{\pi}{6}D^3 = A$

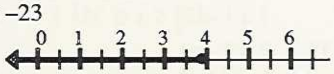
65. $V = 60$, so $60 = 2\sqrt{3S}$
$30 = \sqrt{3S}$
$900 = 3S$
$300 \text{ feet} = S$

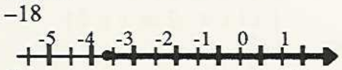
69. $\dfrac{i}{2} = \sqrt{\dfrac{A}{P}} - 1$
$1 + \frac{i}{2} = \sqrt{\dfrac{A}{P}}$
$(1 + \frac{i}{2})^2 = \dfrac{A}{P}$
$P(1 + \frac{i}{2})^2 = A$

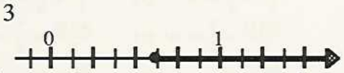
Exercise 2–4

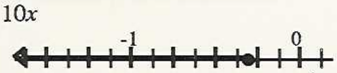
1. $5x + 2 > 3x - 8$
$2x > -10$

$x > -5$ $\{ x \mid x > -5 \}$

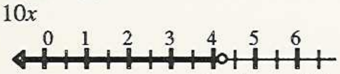
5. $2(3x - 3) \geq 9x + 1$
$6x - 6 \geq 9x + 1$
$-7 \geq 3x$

$-\frac{7}{3} \geq x$ $\{ x \mid x \leq -2\frac{1}{3} \}$

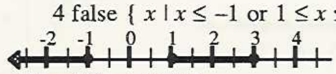
9. $-3(x-2) + 2(x+1) \geq 5(x-3)$
$-3x + 6 + 2x + 2 \geq 5x - 15$
$-6x \geq -23$

$x \leq \frac{23}{6}$ $\{ x \mid x \leq 3\frac{5}{6} \}$

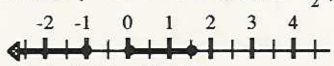
13. $5x + 18 \geq 0$
$5x \geq -18$

$x \geq -\frac{18}{5}$ $\{ x \mid x \geq -3\frac{3}{5} \}$

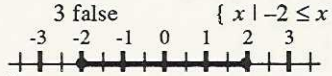
17. $9x \geq 4x + 3$
$5x \geq 3$

$x \geq \frac{3}{5}$ $\{ x \mid x \geq \frac{3}{5} \}$

21. $12 + 3(3 - 2x) > 4(x + 6)$
$12 + 9 - 6x > 4x + 24$
$-3 > 10x$

$-\frac{3}{10} > x$ $\{ x \mid x < -\frac{3}{10} \}$

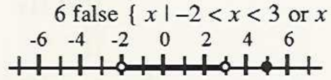
25. $6 - x > 9x - 36$
$42 > 10x$

$\frac{21}{5} > x$ $\{ x \mid x < 4\frac{1}{5} \}$

29. $(x-3)(x+1)(x-1) \leq 0$
CP: -1 true
1 true
3 true
TP: -2 true
0 false
2 true
4 false $\{ x \mid x \leq -1$ or $1 \leq x \leq 3 \}$


33. $r(2r-3)(r+1) \leq 0$
CP: -1 true
0 true
$\frac{3}{2}$ true
TP: -2 true
$-\frac{1}{2}$ false
1 true
2 false $\{ r \mid r \leq -1$ or $0 \leq r \leq 1\frac{1}{2} \}$


37. $(x^2 - 4x + 4)(x^2 - 4) \leq 0$
$(x-2)^3(x+2) \leq 0$
CP: -2 true, 2 true
TP: -3 false
0 true
3 false $\{ x \mid -2 \leq x \leq 2 \}$


41. $\dfrac{3}{x} > 0$
CP: 0
TP: -1 false
1 true $\{ x \mid x > 0 \}$


45. $\dfrac{x^2 - 10x + 25}{x^2 - x - 6} \leq 0$
$\dfrac{(x-5)^2}{(x-3)(x+2)} \leq 0$
CP: -2, 3, 5 true
TP: -3 false
0 true
4 false
6 false $\{ x \mid -2 < x < 3$ or $x = 5 \}$


49. $\dfrac{x}{x+1} - \dfrac{2}{x+3} \leq 1$
$\dfrac{x(x+3) - 2(x+1)}{(x+1)(x+3)}$
$- \dfrac{(x+1)(x+3)}{(x+1)(x+3)} \leq 0$
$\dfrac{-3x - 5}{(x+1)(x+3)} \leq 0$
CP: -3, $-\frac{5}{3}$ true, -1
TP: -4 false
-2 true
$-1\frac{1}{10}$ false
0 true $\{x \mid -3 < x \leq -1\frac{2}{3}$ or $x > -1\}$


53. $\sqrt{9 - 2x}$
$9 - 2x \geq 0$
$-2x \geq -9$
$x \leq \frac{9}{2}$

57. $\sqrt{4x^2 - 4x - 3}$
$4x^2 - 4x - 3 \geq 0$
$(2x-3)(2x+1) \geq 0$
Critical Points: $-\frac{1}{2}$, $1\frac{1}{2}$
Test Points: $-1, 0, 2$
True at -1 and 2.
$x \leq -\frac{1}{2}$ or $x \geq 1\frac{1}{2}$

61. $P = 2\ell + 2w$ (Perimeter is twice the length plus twice the width.)
$P = 2(30) + 2W$
$P = 60 + 2W$
We want $60 + 2w < 100$
$2w < 40$
$w < 20.$
We also want $w > 0$, so we require $0 < W < 20$

65. $\dfrac{1}{x} + \dfrac{1}{x+10} \geq \dfrac{1}{40}$; $\dfrac{(x+10)+x}{x(x+10)} \geq \dfrac{1}{40}$; $\dfrac{2x+10}{x(x+10)} - \dfrac{1}{40} \geq 0$;
$\dfrac{(80x+400) - x(x+10)}{40x(x+10)} \geq 0$; $\dfrac{-x^2 + 70x + 400}{40x(x+10)} \geq 0$;
Critical points are $35 \pm 5\sqrt{65}$ ($\approx 75, -5.3$)true , 0, and -10.

Since $x > 0$ we check the intervals determined by 0 and $35 + 5\sqrt{65}$, using test points of, say 1 and 80. We find $0 < x \leq 35 + 5\sqrt{65}$.

69.

	L	W	$P = 2l + 2w$	$A = lw$	$\frac{16A}{P^2}$	
a.	100'	50'	300	5000	0.89	Conforms
b.	200'	50'	500	10000	0.64	Conforms
c.	300'	50'	700	15000	0.49	Conforms
d.	400'	50'	900	20000	0.395	NO
e.	500'	50'	1100	25000	0.33	NO
f.	600	50'	1300	30000	0.28	NO

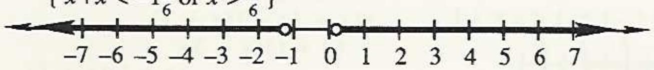
Exercise 2–5

1. $|5x| = 8$
 $5x = 8$ or $5x = -8$
 $x = \frac{8}{5}$ or $x = -\frac{8}{5}$
 $\{\pm\frac{8}{5}\}$

5. $|3 - 2x| = 5$
 $3 - 2x = 5$ or $3 - 2x = -5$
 $-2 = 2x$ or $8 = 2x$
 $-1 = x$ or $4 = x$
 $\{-1, 4\}$

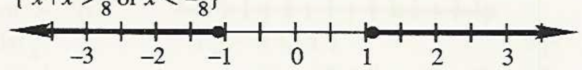
9. $\left|\frac{3x-5}{4}\right| = 1$
 $\frac{3x-5}{4} = 1$ or $\frac{3x-5}{4} = -1$
 $3x - 5 = 4$ or $3x - 5 = -4$
 $3x = 9$ or $3x = 1$
 $x = 3$ or $x = \frac{1}{3}$
 $\{\frac{1}{3}, 3\}$

13. $|3 + 6x| > 4$
 $3 + 6x > 4$ or $3 + 6x < -4$
 $6x > 1$ or $6x < -7$
 $x > \frac{1}{6}$ or $x < -\frac{7}{6}$
 $\{x \mid x < -1\frac{1}{6}$ or $x > \frac{1}{6}\}$



17. $|2 - 5x| < -3$
 No solution since $|2 - 5x| \geq 0$.
 Φ

21. $\left|3x - \frac{x}{3}\right| > 3$
 $\left|\frac{9x}{3} - \frac{x}{3}\right| > 3$
 $\left|\frac{8x}{3}\right| > 3$
 $\frac{8x}{3} > 3$ or $\frac{8x}{3} < -3$
 $8x > 9$ or $8x < -9$
 $x > \frac{9}{8}$ or $x < -\frac{9}{8}$
 $\{x \mid x > \frac{9}{8}$ or $x < -\frac{9}{8}\}$



25. $|3x| > 22$
 $3x > 22$ or $3x < -22$
 $x > \frac{22}{3}$ or $x < -\frac{22}{3}$
 $\{x \mid x > 7\frac{1}{3}$ or $x < -7\frac{1}{3}\}$

29. $\left|\frac{x-2}{4}\right| > 9$
 $\frac{x-2}{4} > 9$ or $\frac{x-2}{4} < -9$
 $x - 2 > 36$ or $x - 2 < -36$
 $x > 38$ or $x < -34$
 $\{x \mid x < -34$ or $x > 38\}$

33. $|5 - 2x| = 25$
 $5 - 2x = 25$ or $5 - 2x = -25$
 $-10 = x$ or $15 = x$
 $\{-10, 15\}$

37. $|3x| < 22$
 $-22 < 3x < 22$
 $-\frac{22}{3} < x < \frac{22}{3}$
 $\{x \mid -7\frac{1}{3} < x < 7\frac{1}{3}\}$

41. $\left|\frac{x-2}{4}\right| < 9$
 $-9 < \frac{x-2}{4} < 9$
 $-36 < x - 2 < 36$
 $-34 < x < 38$
 $\{x \mid -34 < x < 38\}$

45. $5 > |6x - 3|$
 $5 > 6x - 3 > -5$
 $8 > 6x > -2$
 $\frac{4}{3} > x > -\frac{1}{3}$
 $\{x \mid -\frac{1}{3} < x < 1\frac{1}{3}\}$

49. $\left|\frac{2x-3}{4}\right| \leq 17$
 $-17 \leq \frac{2x-3}{4} \leq 17$
 $-68 \leq 2x - 3 \leq 68$
 $-65 \leq 2x \leq 71$
 $-\frac{65}{2} \leq x \leq \frac{71}{2}$
 $\{x \mid -32\frac{1}{2} \leq x \leq 35\frac{1}{2}\}$

53. $2 < |-x|$
 $|-x| > 2$
 $-x > 2$ or $-x < -2$
 $x < -2$ or $x > 2$
 $\{x \mid x < -2$ or $x > 2\}$

57. $|6\frac{1}{2} - w| = \frac{7}{8}$
 $6\frac{1}{2} - w = \frac{7}{8}$ or $6\frac{1}{2} - w = -\frac{7}{8}$
 $\frac{13}{2} - \frac{7}{8} = w$ or $\frac{13}{2} + \frac{7}{8} = w$
 $\frac{4(13)}{8} - \frac{7}{8} = w$ or $\frac{4(13)}{8} + \frac{7}{8} = w$
 $\frac{45}{8} = w$ or $\frac{59}{8} = w$
 The dimension y is $5\frac{5}{8}$ and the dimension x is $7\frac{3}{8}$ inches.

Chapter 2 Review

1. $\frac{3}{5}x - 4 = 2 - \frac{3}{4}x$
 $\frac{3}{5}x + \frac{3}{4}x = 6$
 $\frac{27}{20}x = 6$
 $27x = 6 \cdot 20$
 $x = \frac{120}{27} = \frac{40}{9}$ $\{4\frac{4}{9}\}$

3. $-2[\frac{1}{2} - 2(5 - x) + 2] - \frac{3}{2}x = 0$
 $-2[\frac{1}{2} - 10 + 2x + 2] - \frac{3}{2}x = 0$
 $-1 + 20 - 4x - 4 - \frac{3}{2}x = 0$
 $15 - 4x - \frac{3}{2}x = 0$
 $15 = 4x + \frac{3}{2}x$
 $30 = 8x + 3x$
 $30 = 11x$
 $\frac{30}{11} = x$ $\{2\frac{8}{11}\}$

5. $x - \frac{3}{8}x = \frac{1}{4}x$
 $8x - 3x = 2x$
 $5x = 2x$
 $3x = 0$
 $x = 0$ $\{0\}$

7. $11.4 - 3.5x - \sqrt{2}[9.2 - 1.5(\frac{3}{8}x - 5.3) - x] = 0$
 $11.4 - 3.5x - \sqrt{2}(9.2 - (0.5625x - 7.95) - x) = 0$

$11.4 - 3.5x - \sqrt{2}(-1.5625x + 17.15) = 0$
$11.4 - 3.5x - (-1.5625\sqrt{2}x + 17.15\sqrt{2}) = 0$
$-3.5x + 1.5625\sqrt{2}x - 17.15\sqrt{2} + 11.4 = 0$
$-1.29029x - 12.8538 = 0$
$-1.29029x = 12.8538$
$x \approx -9.9619$

9. $R = W - k(2c + b)$; for b
$R = W - 2kc - kb$
$kb = W - 2kc - R$
$b = \dfrac{W - 2kc - R}{k}$

11. $\dfrac{x + 2y}{3 - 2y} = x$; for x
$x + 2y = x(3 - 2y)$
$x + 2y = 3x - 2xy$
$2y = 2x - 2xy$
$2y = x(2 - 2y)$
$x = \dfrac{2y}{2 - 2y} = \dfrac{y}{1 - y}$

13. x = amount invested at 7%; then $8000 - x$ = amount invested at 5%. 7% of x plus 5% of $(8000 - x)$ is 471
$0.07x + 0.05(8000 - x) = 471$
$0.07x + 400 - 0.05x = 471$
$0.02x = 71$
$x = \dfrac{71}{0.02} = 3550$
$8000 - x = 4450$
Thus \$3550 was invested at 7% and \$4550 was invested at 5%.

15. Let x be the amount invested at 5%. Then $5000 - x$ was invested at 9%. Income from 9% investment is $0.09(5000 - x)$; income from 5% investment is $0.05x$. The difference in the investments is \$44, with the 9% investment the larger.
Thus: Investment at 9% – Investment at 5% is \$44
$0.09(5000 - x) - 0.05x = 44$
$450 - 0.09x - 0.05x = 44$
$406 = 0.14x$
$\dfrac{406}{0.14} = x$
$2900 = x$
Thus \$2,900 was invested at 5%, and $5000 - 2900 =$ \$2,100 was invested at 9%.

17. Let x be the amount of 55% copper. Then the total amount, after mixing, will be $x + 15$ tons. The amount of copper in the final mixture comes from the 40% and 55% alloys (an alloy is a mixture of metals). The total amount of copper will be 45% of $x + 15$, and it comes from 40% of the 15 tons, and 55% of the x tons. Thus:
$0.45(x + 15) = 0.40(15) + 0.55x$
$0.45x + 6.75 = 6 + 0.55x$
$0.75 = 0.10x$
$\dfrac{0.75}{0.10} = 7.5 = x$
Thus 7.5 tons of 55% copper should be mixed with the existing 15 tons of 40% copper. The resulting mixture will be $7.5 + 15 = 22.5$ tons of alloy which is 40% copper.

19. We want the speed of the current, so let x be that speed. Downstream the rate of the boat is $12 + x$ (speed of boat + speed of current); upstream the rate is $12 - x$ (speed of boat, less the speed of the current). Since $RT = D$, $T = \dfrac{D}{R}$. (We think in terms of time since we have two trips made in equal time.)
Time for trip downstream: $T = \dfrac{D}{R} = \dfrac{20}{12 + x}$
Time for trip upstream: $T = \dfrac{D}{R} = \dfrac{14}{12 - x}$
These two times are equal: $\dfrac{20}{12 + x} = \dfrac{14}{12 - x}$
$20(12 - x) = 14(12 + x)$
$240 - 20x = 168 + 14x$
$72 = 24x$
$\dfrac{72}{24} = x$
$3 = x$
Thus the speed of the stream is 3 mph.

21. $2x^2 - 7x - 30 = 0$
$(2x + 5)(x - 6) = 0$
$2x + 5 = 0$ or $x - 6 = 0$
$x = -\dfrac{5}{2}$ or $x = 6$

23. $(2x - 5)^2 = 40 - 16x$
$4x^2 - 20x + 25 = 40 - 16x$
$4x^2 - 4x - 15 = 0$
$(2x - 5)(2x + 3) = 0$
$2x - 5 = 0$ or $2x + 3 = 0$
$x = \dfrac{5}{2}$ or $x = -\dfrac{3}{2}$

25. $27x^2 - 40 = 0$
$27x^2 = 40$
$x^2 = \dfrac{40}{27}$
$x = \pm\sqrt{\dfrac{40}{27}} = \pm\dfrac{\sqrt{40}}{\sqrt{27}}$
$x = \pm\dfrac{2\sqrt{10}}{3\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$
$x = \pm\dfrac{2\sqrt{30}}{9}$

27. $4(x + 1)^2 = 8$
$(x + 1)^2 = 2$
$x + 1 = \pm\sqrt{2}$
$x = -1 \pm \sqrt{2}$

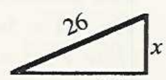
$$\boxed{\text{If } ax^2 + bx + c = 0 \text{ then } x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$$

29. $5y^2 - 15y - 6 = 0$ $a = 5, b = -15, c = -6$.
$y = \dfrac{-(-15) \pm \sqrt{(-15)^2 - 4(5)(-6)}}{2(5)} = \dfrac{15 \pm \sqrt{345}}{10}$

31. $(z + 4)(z - 1) = 5z + 4$
$z^2 + 3z - 4 = 5z + 4$
$z^2 - 2z - 8 = 0$
$z = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$ $a = 1, b = -2, c = -8$.
$z = \dfrac{2 \pm \sqrt{36}}{2} = \dfrac{2 \pm 6}{2}$
$z = -2$ or 4

33. $3x^2 - 8x - 12$
The roots of $3x^2 - 8x - 12 = 0$ are $\dfrac{4 \pm 2\sqrt{13}}{3}$ (using the quadratic formula), so
$3x^2 - 8x - 12 = 3\left(x - \dfrac{4 + 2\sqrt{13}}{3}\right)\left(x - \dfrac{4 - 2\sqrt{13}}{2}\right)$.

35. $x^2 + (x + 14)^2 = 26^2$
$x^2 + x^2 + 28x + 196 = 676$
$2x^2 + 28x - 480 = 0$
$x^2 + 14x - 240 = 0$
$(x - 10)(x + 24) = 0$
$x - 10 = 0$ or $x + 24 = 0$
$x = 10$ or -24.



Since -24 is not a valid value for the length of a side of a triangle, $x = 10$. The length of the longer side is $10 + 14 = 24$.

37. Total Cost = Total Revenue
$C = R$
$250x^2 - 24000 = 200x - 2000$
$250x^2 - 200x - 22000 = 0$
$5x^2 - 4x - 440 = 0$ Divide each term by 50.
$x = \dfrac{4 \pm \sqrt{8816}}{10} \approx 10$ or -9.
Since we assume only a nonnegative number of units may be sold the break even point is 10 units.

39. Let T be the required time for the aircraft to fly 500 miles in no wind.
Then $T + 1$ is the time for flying into the wind. Using $d = rt$ we can express d for each flight (300 miles) and t for each flight (T and $T + 1$). We do not have a way to express rate yet. This indicates solving for r: $r = \frac{d}{t}$, and expressing rate for each flight:

Rate in flight with no wind: $\frac{d}{t} = \frac{300}{T}$

Rate in flight into the wind: $\frac{d}{t} = \frac{300}{T + 1}$

We know that the rate into the wind is 25 mph slower than the rate with no wind. Thus

Rate with no wind less rate into wind = 25

$\frac{300}{T} - \frac{300}{T + 1} = 25$

$\frac{12}{T} - \frac{12}{T + 1} = 1$ Divide each term by 25.

$T(T + 1)\frac{12}{T} - T(T + 1)\frac{12}{T + 1} = T(T + 1)(1)$

$12(T + 1) - 12T = T^2 + T$

$12T + 12 - 12T = T^2 + T$

$0 = T^2 + T - 12$

$(T + 4)(T - 3) = 0$

$T = -4$ or 3 ; we assume $T > 0$, so it takes 3 hours to fly the 500 miles in no wind.

41. $\frac{2x - 5}{x^2 + 8x + 12}$

$x^2 + 8x + 12 \neq 0$

$(x + 2)(x + 6) \neq 0$

$x + 2 \neq 0$ and $x + 6 \neq 0$

$x \neq -2$ and $x \neq -6$

43. $(x - 3)^2 - 8(x - 3) - 20 = 0$

$u^2 - 8u - 20 = 0$ Substitute u for $x - 3$.

$(u + 2)(u - 10) = 0$

$u = -2$ or $u = 10$

$x - 3 = -2$ or $x - 3 = 10$

$x = 1$ or $x = 13$

45. $x^{3/2} - 9x^{3/4} + 8 = 0$

$u^2 - 9u + 8 = 0$ Let $u = x^{3/4}$, so $u^2 = (x^{3/4})^2 = x^{3/2}$.

$(u - 8)(u - 1) = 0$

$u = 8$ or $u = 1$

$x^{3/4} = 8$ or $x^{3/4} = 1$

$(x^{3/4})^{4/3} = 8^{4/3}$ or $(x^{3/4})^{4/3} = 1^{4/3}$

$x = (\sqrt[3]{8})^4$ or $x = 1$

$x = 16$ or $x = 1$

47. $\sqrt{x(6x + 5)} = 1$

$x(6x + 5) = 1$ Square both sides.

$6x^2 + 5x - 1 = 0$

$(6x - 1)(x + 1) = 0$

$x = \frac{1}{6}$ or $x = -1$

Both solutions check.

49. $\sqrt{5w + 1} = 5w - 19$

$(\sqrt{5w + 1})^2 = (5w - 19)^2$

$5w + 1 = 25w^2 - 190w + 361$

$0 = 25w^2 - 195w + 360$

$0 = 5w^2 - 39w + 72$ Divide each member by 5.

$0 = (w - 3)(5w - 24)$

$w = 3$ or $w = \frac{24}{5}$.

The value 3 will not check, so the answer is $w = \frac{24}{5} = 4\frac{4}{5}$.

51. $\sqrt[4]{2y - 3} = 2$

$(\sqrt[4]{2y - 3})^4 = 2^4$

$2y - 3 = 16$

$y = \frac{19}{2}$. Thus solution checks.

53. $r = \sqrt{\frac{A}{\pi} - AR^2}$; for A

$r^2 = \left(\sqrt{\frac{A}{\pi} - AR^2}\right)^2$

$r^2 = \frac{A}{\pi} - AR^2$

$\pi r^2 = A - A\pi R^2$

$\pi r^2 = A(1 - \pi R^2)$

$\frac{\pi r^2}{1 - \pi R^2} = A$

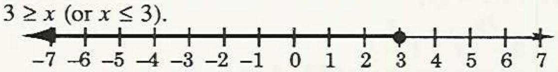
55. $3(x - 1) + 2(3 - 2x) \geq 5(x - 3)$

$3x - 3 + 6 - 4x \geq 5x - 15$

$-x + 3 \geq 5x - 15$

$18 \geq 6x$

$3 \geq x$ (or $x \leq 3$).



57. $12 + \frac{5}{2}(3 - \frac{2}{3}x) < -4(x - 6)$

$12 + \frac{15}{2} - \frac{5}{3}x < -4x + 24$

$4x - \frac{5}{3}x < 12 - \frac{15}{2}$

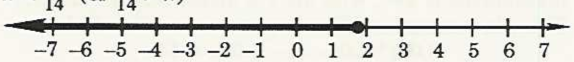
$\frac{12}{3}x - \frac{5}{3}x < \frac{24}{2} - \frac{15}{2}$

$\frac{7}{3}x < \frac{9}{2}$

$7x < \frac{27}{2}$

$x < \frac{1}{7} \cdot \frac{27}{2}$

$x < \frac{27}{14}$ (or $\frac{27}{14} > x$)



59. $w^2 - 1 < \frac{7}{12}w$

Critical Points: $w^2 - 1 = \frac{7}{12}w$

$12w^2 - 12 = 7w$

$12w^2 - 7w - 12 = 0$

$(3x - 4)(4x + 3) = 0$

$x = \frac{4}{3}$ or $-\frac{3}{4}$ (Not part of solution set.)

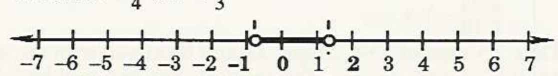
Test Points: $w^2 - 1 < \frac{7}{12}w$

-1: $(-1)^2 - 1 < \frac{7}{12}(-1)$; $0 < -\frac{7}{12}$ FALSE

0: $0^2 - 1 < \frac{7}{12}0$; $-1 < 0$ TRUE

2: $2^2 - 1 < \frac{7}{12}(2)$; $3 < \frac{7}{6}$ FALSE

Solution: $-\frac{3}{4} < w < \frac{4}{3}$



61. $(x^2 - 6x + 9)(x^2 - 1) \leq 0$

Critical Points: $(x^2 - 6x + 9)(x^2 - 1) = 0$

$(x - 3)^2(x - 1)(x + 1) = 0$

$x = 3, \pm 1$. (Part of the solution set.)

Test Points: $(x^2 - 6x + 9)(x^2 - 1) \leq 0$

-2: $((-2)^2 - 6(-2) + 9)((-2)^2 - 1) \leq 0$

$75 \leq 0$ FALSE

0: $(0^2 - 6(0) + 9)(0^2 - 1) \leq 0$

$-9 \leq 0$ TRUE

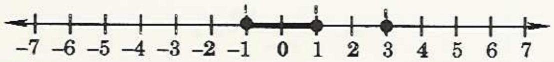
2: $(2^2 - 6(2) + 9)(2^2 - 1) \leq 0$

$3 \leq 0$ FALSE

4: $(4^2 - 6(4) + 9)(4^2 - 1) \leq 0$

$15 \leq 0$ FALSE

Solution: $-1 \leq x \leq 1$ or $x = 3$.



63. $\dfrac{x-1}{x-3} \ge 0$

Critical Points: $\dfrac{x-1}{x-3} = 0$

$x - 3 = 0$ (zeros of denominators are critical points.)

$x = 3$ (not part of solution)

$x - 1 = 0$

$x = 1$ (part of solution)

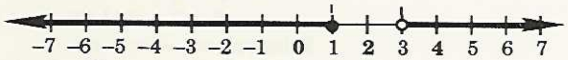
Test Points: $\dfrac{x-1}{x-3} \ge 0$

0: $\dfrac{0-1}{0-3} \ge 0\,;\ \dfrac{1}{3} \ge 0$ TRUE

2: $\dfrac{2-1}{2-3} \ge 0\,;\ -\dfrac{1}{3} \ge 0$ FALSE

4: $\dfrac{4-1}{4-3} \ge 0\,;\ 3 \ge 0$ TRUE

Solution: $x \le 1$ or $x > 3$.



65. $\dfrac{x+3}{x^2 - x - 6} < 0$

Critical Points: $\dfrac{x+3}{x^2 - x - 6} = 0$

$x^2 - x - 6 = 0$ Zeros of denominator.

$(x+2)(x-3) = 0$

$x = -2$ or 3 (Not part of solution.)

$x + 3 = 0$

$x = -3$ (Not part of solution.)

Test Points: $\dfrac{x+3}{x^2 - x - 6} < 0$

−4: $\dfrac{(-4)+3}{(-4)^2 - (-4) - 6} < 0$

$-0.07 < 0$ TRUE

−2.5: $\dfrac{(-2.5)+3}{(-2.5)^2 - (-2.5) - 6} < 0$

$0.18 < 0$ FALSE

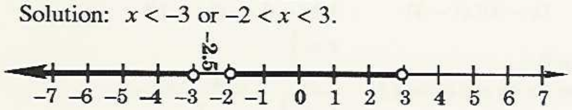
0: $\dfrac{0+3}{0^2 - 0 - 6} < 0$

$-0.5 < 0$ TRUE

4: $\dfrac{4+3}{4^2 - 4 - 6} < 0$

$1.17 < 0$ FALSE

Solution: $x < -3$ or $-2 < x < 3$.



67. $\dfrac{3x}{x-1} - \dfrac{2}{x+3} < 1$

Critical Points: $\dfrac{3x}{x-1} - \dfrac{2}{x+3} = 1$

$x - 1 = 0$ Zeros of denominators are critical points.

$x = 1$

$x + 3 = 0\,;\ x = -3$

$\dfrac{3x(x+3) - 2(x-1)}{(x-1)(x+3)} = 1$

$\dfrac{3x^2 + 7x + 2}{x^2 + 2x - 3} = 1$

$3x^2 + 7x + 2 = x^2 + 2x - 3$

$2x^2 + 5x + 5 = 0$ Solutions are complex, so no critical points here.

The critical points are −3, 1. They are not part of the solution set.

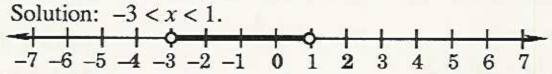
Test Points: $\dfrac{3x}{x-1} - \dfrac{2}{x+3} < 1$

−4: $\dfrac{3(-4)}{-4-1} - \dfrac{2}{-4+3} < 1\,;\ 4.4 < 1$ FALSE

0: $\dfrac{3(0)}{0-1} - \dfrac{2}{0+3} < 1\,;\ -0.7 < 1$ TRUE

2: $\dfrac{3(2)}{2-1} - \dfrac{2}{2+3} < 1\,;\ 5.6 < 1$ FALSE

Solution: $-3 < x < 1$.



69. $\frac{3}{8} - 2x = \frac{3}{4}$ or $\frac{3}{8} - 2x = -\frac{3}{4}$

$3 - 16x = 6$ or $3 - 16x = -6$

$-16x = 3$ or $-16x = -9$

$x = -\frac{3}{16}$ or $x = \frac{9}{16}$

71. $x^2 + 1 = 1$ or $x^2 + 1 = -1$

$x^2 = 0$ or $x^2 = -2$

$x = 0$ or $x = \pm\sqrt{-2} = \pm\sqrt{2}\,i$

73. $x^2 - x = 2$ or $x^2 - x = -2$

$x^2 - x - 2 = 0$ or $x^2 - x + 2 = 0$

$x = -1, 2$ $\quad x = \frac{1}{2}(1 \pm \sqrt{7}i)$

75. $\dfrac{x-2}{4} > 9$ or $\dfrac{x-2}{4} < -9$

$x - 2 > 36$ or $x - 2 < -36$

$x > 38$ or $x < -34$

77. $|-2x - 3| < 5$

$-5 < -2x - 3 < 5$

$-2 < -2x < 8$

$\dfrac{-2}{-2} > \dfrac{-2x}{-2} > \dfrac{8}{-2}$

$1 > x > -4$

$-4 < x < 1$

79. $3x - \frac{4}{5} > 2$ or $3x - \frac{4}{5} < -2$

$3x > \frac{14}{5}$ or $3x < -\frac{6}{5}$

$x > \frac{14}{15}$ or $x < -\frac{2}{5}$

Chapter 2 Test

1. $7x - 4 = 7(4 - x)$

$7x - 4 = 28 - 7x$

$14x = 32$

$x = \frac{32}{14} = \frac{16}{7}$ $\quad \{\frac{16}{7}\}$

3. $3x - \frac{3}{4}x = \frac{1}{4}x + 2$

$12x - 3x = x + 8$ Multiply each term by 4.

$8x = 8$

$x = 1$ $\quad \{\ 1\ \}$

5. $m = -p(Q - x)$; for x

$m = -pQ + px$

$m + pQ = px$

$\dfrac{m + pQ}{p} = x$

7. $\dfrac{x + 2y}{3 - 2y} = x$; for y

$x + 2y = x(3 - 2y)$

$x + 2y = 3x - 2xy$

$2xy + 2y = 3x - x$

$2y(x + 1) = 2x$

$y = \dfrac{2x}{2(x+1)}$

$y = \dfrac{x}{x+1}$

9. Let x be the amount of 80% copper alloy.

Analyzing just the copper: 30% of 28 tons + 80% of x tons gives 50% of $(28 + x)$ tons.

$.3(28) + 0.8x = 0.5(28 + x)$

$x =$ 18.67 tons

18.67 tons of 80% copper alloy should be used.

11. Basic Principle: Rate x Time = Distance

Let x be the speed of the current.
To use the words "in the same time", we focus on the time for each trip:

Time $= \frac{\text{Distance}}{\text{Rate}}$

Time Downstream: $\frac{20}{10 + x}$

Time Upstream: $\frac{15}{10 - x}$

These times are equal, so

$\frac{20}{10 + x} = \frac{15}{10 - x}$

$20(10 - x) = 15(10 + x)$; $x = \frac{10}{7} = 1\frac{3}{7}$ miles per hour for the speed of the current.

13. $10 + \frac{13}{x} = \frac{3}{x^2}$

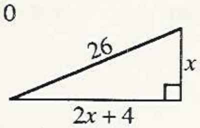
$10x^2 + 13x = 3$ Multiply each term by x^2.
$10x^2 + 13x - 3 = 0$
$(2x + 3)(5x - 1) = 0$
$2x + 3 = 0$ or $5x - 1 = 0$
$x = -\frac{3}{2}$ or $x = \frac{1}{5}$

15. $(3x - 3)^2 = 24$
$3x - 3 = \pm\sqrt{24}$
$3x = 3 \pm 2\sqrt{6}$
$x = 1 \pm \frac{2}{3}\sqrt{6}$

17. $(z - 1)(z + 1) = \frac{1}{3}(-6z - 7)$
$z^2 - 1 = \frac{1}{3}(-6z - 7)$
$3z^2 - 3 = 1(-6z - 7)$
$3z^2 + 6z + 4 = 0$

$z = \frac{-6 \pm \sqrt{6^2 - 4(3)(4)}}{2(3)} = \frac{-6 \pm \sqrt{-12}}{6} = \frac{-6 \pm 2\sqrt{3}\,i}{6} = \frac{-6}{6} \pm \frac{2\sqrt{3}}{6}\,i = -1 \pm \frac{\sqrt{3}}{3}\,i$

19. Let x be the length of the shorter side.
$(2x + 4)^2 + x^2 = 26^2$
$5x^2 + 16x - 660 = 0$
$x = 10$ (Use the quadratic formula, and $x > 0$.) x is 10, so the longer side is $2(10) + 4 = 24$.



21. Principle: Rate of doing work x time = amount of work done.
If printing an edition of the newspaper is "one job" (amount of work done), then the principle is: rate x time = 1, so rate $= \frac{1}{\text{time}}$.

Let x be the time for the faster press to print the edition alone.

Rate for faster press alone: $\frac{1}{x}$

Rate for the slower press alone: $\frac{1}{x + 1}$

Rate for the presses together: $\frac{1}{12}$

Thus $\frac{1}{x} + \frac{1}{x + 1} = \frac{1}{12}$

$(x)(x + 1)(12)\frac{1}{x} + (x)(x + 1)(12)\frac{1}{x + 1} = (x)(x + 1)(12)\frac{1}{12}$

$12(x + 1) + 12x = x(x + 1)$

$0 = x^2 - 23x - 12$
$x \approx -0.5$ or 23.5
Thus the faster press takes about 23.5 hours for the press run alone.

23. $\frac{x - 1}{x^2 - 81}$
$x^2 - 81 \neq 0$
$(x - 9)(x + 9) \neq 0$
$x \neq \pm 9$

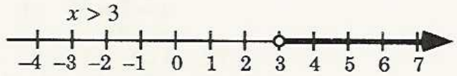
25. $x^3 - 7x^{3/2} - 8 = 0$
$u^2 - 7u - 8 = 0$
Let $u = x^{3/2}$, so $u^2 = (x^{3/2})^2 = x^3$
$(u - 8)(u + 1) = 0$
$u = 8$ or $u = -1$
$x^{3/2} = 8$ or $x^{3/2} = -1$
$x^{3/2} = -1$ does not have a real solution, because $x^{3/2}$ means $(\sqrt{x})^3$, and since $\sqrt{x} \geq 0$, then $(\sqrt{x})^n \geq 0$ for any integer n, including 3.
$(x^{3/2})^{2/3} = 8^{2/3}$

$x^1 = (\sqrt[3]{8})^2 = 2^2 = 4$

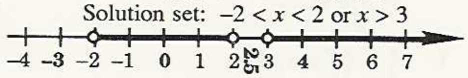
27. $\sqrt{5n + 5} - \sqrt{13 - n} = 2$
$\sqrt{5n + 5} = \sqrt{13 - n} + 2$
$(\sqrt{5n + 5})^2 = (\sqrt{13 - n} + 2)^2$
$5n + 5 = (13 - n) + 4\sqrt{13 - n} + 4$
$6n - 12 = 4\sqrt{13 - n}$
$3n - 6 = 2\sqrt{13 - n}$
$(3n - 6)^2 = (2\sqrt{13 - n})^2$
$9n^2 - 36n + 36 = 4(13 - n)$
$9n^2 - 32n - 16 = 0$
$n = 4$ or $-\frac{4}{9}$

4 checks, but $-\frac{4}{9}$ does not.

29. $4x - 3(2x - 3) < x$
$4x - 6x + 9 < x$
$-3x < -9$
$\frac{-3x}{-3} > \frac{-9}{-3}$
$x > 3$



31. $(x - 3)(x^2 - 4) > 0$
Critical Points: $(x - 3)(x^2 - 4) = 0$
$(x - 3)(x - 2)(x + 2) = 0$
$x = -2, 2, 3$ (These are *not* part of the solution set.)
Test Points: $(x - 3)(x^2 - 4) \geq 0$
-3: $(-3 - 3)((-3)^2 - 4) \geq 0$
$-30 \geq 0$ FALSE
0: $(0 - 3)(0^2 - 4) \geq 0$
$12 \geq 0$ TRUE
2.5: $(2.5 - 3)(2.5^2 - 4) \geq 0$
$-1.1 \geq 0$ FALSE
4: $(4 - 3)(4^2 - 4) \geq 0$
$12 \geq 0$ TRUE
Solution set: $-2 < x < 2$ or $x > 3$



33. $\frac{x - 10}{x - 3} > 2$

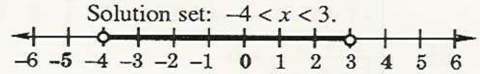
Critical Points: $\frac{x - 10}{x - 3} = 2$

$x - 3 = 0$
$x = 3$ Zero of denominator: not part of the solution set.

$x - 10 = 2(x - 3)$
$-4 = x$ Not part of the solution set.

Test Points: $\frac{x - 10}{x - 3} > 2$

-5: $\frac{-5 - 10}{-5 - 3} > 2$
$1.9 > 2$ FALSE
0: $\frac{0 - 10}{0 - 3} > 2$
$3.3 > 2$ TRUE
4: $\frac{4 - 10}{4 - 3} > 2$
$-6 > 2$ FALSE
Solution set: $-4 < x < 3$.



35. $5 - 2x = 8$ or $5 - 2x = -8$
$-2x = 3$ or $-2x = -13$
$x = -\frac{3}{2}$ or $x = \frac{13}{2}$

37. $\frac{2 - 3x}{4} > 2$ or $\frac{2 - 3x}{4} < -2$
$2 - 3x > 8$ or $2 - 3x < -8$
$-3x > 6$ or $-3x < -10$
$x < -2$ or $x > \frac{10}{3}$

39. $|2x - 3| < 3$
$-3 < 2x - 3 < 3$
$0 < 2x < 6$
$0 < x < 3$

41. $\frac{2x - 3}{3} + \frac{5 - x}{4} = 1 - \frac{2x + 3}{6}$

$24 \cdot \frac{2x - 3}{3} + 24 \cdot \frac{5 - x}{4}$
$= 24 \cdot 1 - 24 \cdot \frac{2x + 3}{6}$
$8(2x - 3) + 6(5 - x) = 24 - 4(2x + 3)$
$10x + 6 = -8x + 12$
$x = \frac{1}{3}$

43. $\frac{2}{x + 1} + \frac{2(x + 2)}{x} = 3$

$x(x + 1)\frac{2}{x + 1} + x(x + 1)\frac{2(x + 2)}{x} = 3x(x + 1)$

$2x + 2(x + 1)(x + 2) = 3x(x + 1)$
$x^2 - 5x - 4 = 0$
$x = \frac{5}{2} \pm \frac{\sqrt{41}}{2}$

45. $\frac{5}{2x - 3} = \frac{4}{x + 1}$
$5(x + 1) = 4(2x - 3)$
$17 = 3x$
$\frac{17}{3} = x$

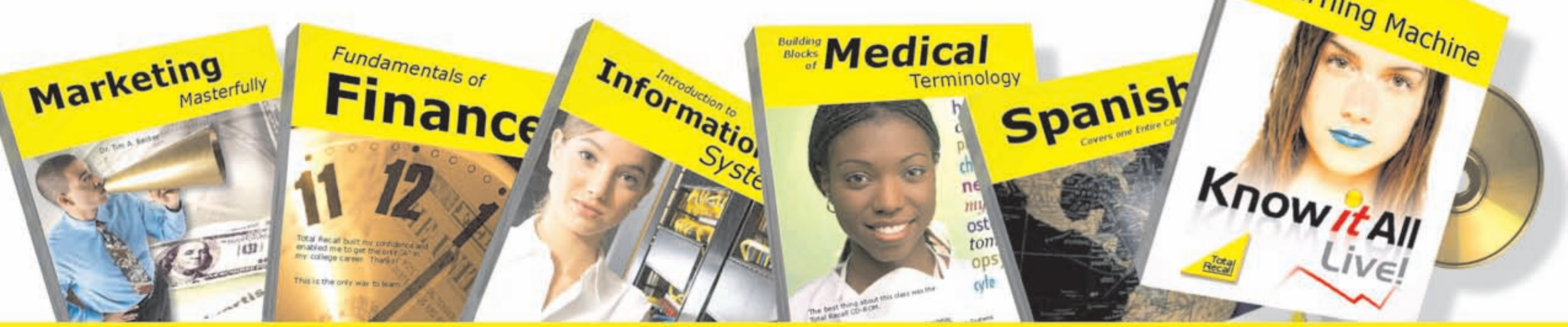
47. $\frac{2x - 5}{x + 2} = 3$
$2x - 5 = 3(x + 2)$
$-11 = x$

How will you learn those terms and definitions? Still trying the **hard** way? Memory studies prove that you will forget **80%** of what you learned that way.

Books are a great reference. But when it comes to learning those important terms, why not make it **easy** on yourself with **exciting, multimedia study guides** from Total Recall Learning, where you will truly **remember everything** you learn!

Download the latest Total Recall study guides from www.FreeeloadPress.com to ensure your success in school or college. Getting an "**A**" has never been so **easy** and so much **fun**!

Information is broken down into **small**, easy-to-digest **pieces**. You will never want to learn any other way again! And with ***Know it All Live!*** you can even prepare your very own multimedia flashcards! For an even greater selection of courses visit www.TotalRecallLearning.com. Make learning **easy**!



www.TotalRecallLearning.com

Exercise 3–1

Answers to problems 1–16 will vary. We solve for y and select values of x which produce integer values of y (for convenience).

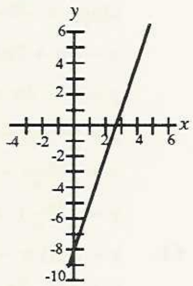
1. $y = 3x - 8$
   $x = 0$: $y = 3(0) - 8 = -8$ $(0, -8)$
   $x = 1$: $y = 3(1) - 8 = -5$ $(1, -5)$
   $x = 2$: $y = 3(2) - 8 = -2$ $(2, -2)$

5. $x = y + 2$
   $y = x - 2$
   $(0, -2)$, $(1, -1)$, $(2, 0)$

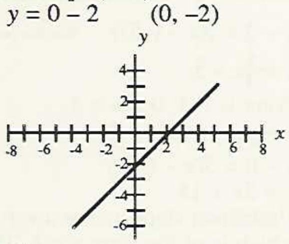
9. $x = y$
   $(1, 1)$, $(2, 2)$, $(3, 3)$

13. $\frac{1}{2}x - \frac{1}{3}y = 1$
   $-\frac{1}{3}y = -\frac{1}{2}x + 1$
   $y = \frac{3}{2}x - 3$
   $(-2, -6)$, $(0, -3)$, $(2, 0)$

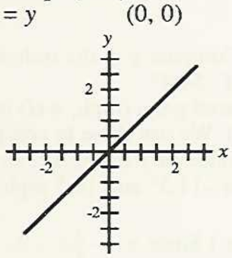
17. $y = 3x - 8$
   x–intercept (y=0):
   $0 = 3x - 8$
   $x = \frac{8}{3}$ $(2\frac{2}{3}, 0)$
   y–intercept (x=0):
   $y = 0 - 8$
   $y = -8$ $(0, -8)$



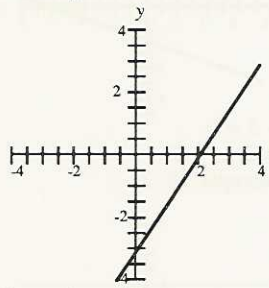
21. $x = y + 2$
   $y = x - 2$
   x–intercept (y=0):
   $x = 2 + 0$ $(2, 0)$
   y–intercept (x=0):
   $y = 0 - 2$ $(0, -2)$



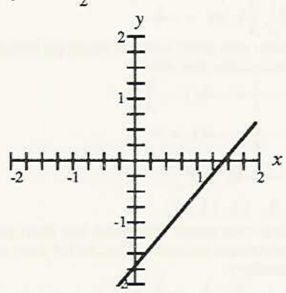
25. $x = y$
   $y = x$
   x–intercept (y=0):
   $x = 0$ $(0, 0)$
   y–intercept (x=0):
   $0 = y$ $(0, 0)$



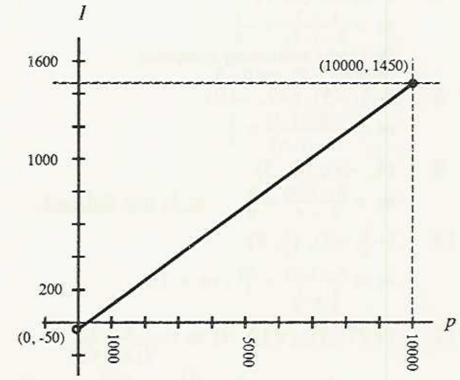
29. $\frac{1}{2}x - \frac{1}{3}y = 1$
   $y = \frac{3x - 6}{2}$
   x–intercept (y=0):
   $\frac{1}{2}x - 0 = 1$
   $x = 2$ $(2, 0)$
   y–intercept (x=0):
   $y = \frac{0 - 6}{2} = -3$ $(0, -3)$



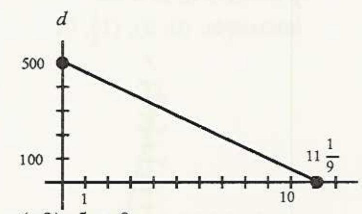
33. $\sqrt{3}x - \sqrt{2}y = \sqrt{6}$
   $y = \frac{\sqrt{6}x - 2\sqrt{3}}{2}$
   x–intercept (y=0):
   $\sqrt{3}\,x - 0 = \sqrt{6}$
   $x = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$ $(\sqrt{2}, 0)$
   y–intercept (x=0):
   $y = \frac{0 - 2\sqrt{3}}{2}$ $(0, -\sqrt{3})$



37. $I = 0.15p - 50$; $0 \le p \le 10{,}000$
   I–intercept (p=0): $I = 0 - 50$ $(0, -50)$
   p–intercept (I=0): $0 = 0.15p - 50$
   $p = \frac{50}{0.15} = 333\frac{1}{3}$ $(333\frac{1}{3}, 0)$
   At p=0 plot $(0, -50)$. At p=10000 plot $(10000, 1450)$.



41. $d = -45t + 500$
   The auto reaches its destination when $d = 0$ (the d–axis intercept).
   Intercepts are $(0, 500)$ and $(11\frac{1}{9}, 0)$.



45. $\left(\frac{1 + (-3)}{2}, \frac{5 + 9}{2}\right) = (-1, 7)$

49. $\left(\frac{\frac{1}{2} + 3\frac{1}{2}}{2}, \frac{-4 + 1}{2}\right) = (2, -1\frac{1}{2})$

53. $\left(\frac{-3 + (-2)}{2}, \frac{4 + 8}{2}\right) = (-2\frac{1}{2}, 6)$

57. $\sqrt{(2 - (-3))^2 + (-1 - 4)^2} = \sqrt{25 + 25}$
   $= \sqrt{25(2)} = 5\sqrt{2}$
   Be careful when a value is negative.
   $(2 - (-3))$ is $2 + 3 = 5$.

61. $\sqrt{(8 - 8)^2 + (-3 - (-4))^2} = \sqrt{0 + 1} = 1$

65. $\sqrt{(0 - 3)^2 + (3 - 0)^2} = \sqrt{9 + 9} = \sqrt{18}$
   $= 3\sqrt{2}$

69. $\sqrt{(3 - (-1))^2 + (\frac{1}{5} - \frac{3}{5})^2} = \sqrt{4^2 + (\frac{2}{5})^2}$
   $= \sqrt{\frac{16(25)}{25} + \frac{4}{25}} = \sqrt{\frac{404}{25}} = \frac{\sqrt{4(101)}}{\sqrt{25}}$
   $= \frac{2\sqrt{101}}{5}$

73. $\sqrt{(2a - (-a))^2 + (-b - 5b)^2} = \sqrt{(3a)^2 + (-6b)^2}$
   $= \sqrt{9a^2 + 36b^2} = \sqrt{9(a^2 + 4b^2)} = 3\sqrt{a^2 + 4b^2}$

77. Let (x, y) be a point equidistant from these two points; the distance from (x, y) to $(1, 2)$ is $\sqrt{(x - 1)^2 + (y - 2)^2}$, and from (x, y) to $(9, 8)$ is $\sqrt{(x - 9)^2 + (y - 8)^2}$. These distances are equal, so $\sqrt{(x - 1)^2 + (y - 2)^2} = \sqrt{(x - 9)^2 + (y - 8)^2}$, and squaring both sides produces $(x - 1)^2 + (y - 2)^2 = (x - 9)^2 + (y - 8)^2$, which reduces to $y = -\frac{4}{3}x + \frac{35}{3}$, a straight line.

81. $3a - 2 = 3$ so $a = \frac{5}{3}$. $5b + 7 - a = -6$, so $b = \frac{a - 13}{5}$
   $= \frac{1}{5} \cdot (\frac{5}{3} - 13) = -\frac{34}{15}$.

85. (a) $d = |x_2 - x_1| + |y_2 - y_1|$
   (b) Taxicab distance is always longer unless the two points lie on the same horizontal or vertical line, in which case they are the same.

89. The second line is $a_2x + b_2y + c_2 = 0$; $a_2 = ka_1$, $b_2 = kb_1$, $c_2 = kc_1$, so the second line is also $ka_1x + kb_1y + kc_1 = 0$, and since $k \ne 0$ we can divide each term by it obtaining $a_1x + b_1y + c_1 = 0$, which is the first line.

Exercise 3-2

1. $(-3, 2), (5, 1)$
$m = \frac{1-2}{5-(-3)} = -\frac{1}{8}$
Be careful subtracting a negative.
Note $5-(-3)$, *not* $5-3$.

5. $(-3, -5), (-7, -10)$
$m = \frac{-10-(-5)}{-7-(-3)} = \frac{5}{4}$

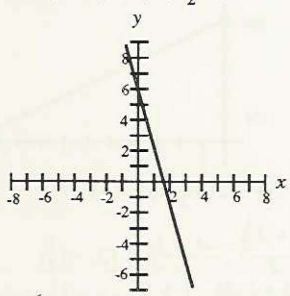
9. $(4, -3), (4, 5)$
$m = \frac{5-(-3)}{4-4} = \frac{8}{0}$ <u>m is not defined.</u>

13. $(-\frac{1}{2}, -2), (\frac{1}{2}, 8)$
$m = \frac{8-(-2)}{\frac{1}{2}+\frac{1}{2}} = \frac{10}{1}$; $m = 10$

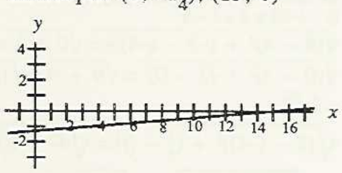
17. $(\sqrt{27}, 3), (\sqrt{12}, 9)$ $m = \frac{9-3}{\sqrt{12}-\sqrt{27}}$
$= \frac{6}{2\sqrt{3}-3\sqrt{3}} = \frac{6}{-\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$

21. Use (1, 1) and (2, –1):
$m = \frac{-1-1}{2-1} = \frac{-2}{1} = -2.$

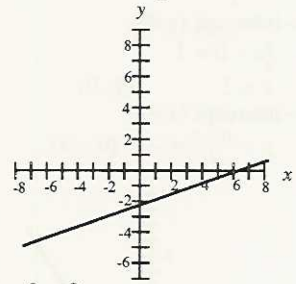
25. $y = 6 - 4x$
$y = -4x + 6$; $m = -4$
intercepts: $(0, 6), (1\frac{1}{2}, 0)$



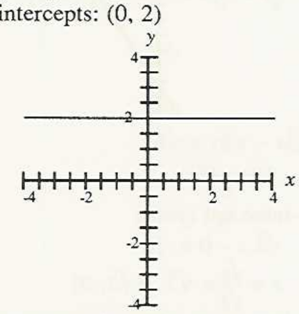
29. $4y = \frac{1}{3}x - 5$
$y = \frac{1}{12}x - \frac{5}{4}$; $m = \frac{1}{12}$
intercepts: $(0, -1\frac{1}{4}), (15, 0)$



33. $\frac{1}{3}x - \frac{5}{6}y = 2$
$y = \frac{2}{5}x - \frac{12}{5}$; $m = \frac{2}{5}$
intercepts: $(0, -2\frac{2}{5}), (6, 0)$



37. $y - 2 = 0$
$y = 2$; $m = 0$
intercepts: (0, 2)



To find the equation of a straight line you must have two pieces of information. (1) One point and (2) either another point or the slope.

41. $(2\frac{1}{4}, \frac{3}{4})$, $m = -4$
Given one point and the slope go directly to the point-slope equation.
$y - \frac{3}{4} = -4(x - \frac{9}{4})$
$y - \frac{3}{4} = -4x + 9$
$y = -4x + 9\frac{3}{4}$

45. $(-3, 1), (5, 4)$
Given two points find slope first *then* go to the point-slope equation. Use either point in this equation.
$m = \frac{4-1}{5-(-3)} = \frac{3}{8}$
$y - 1 = \frac{3}{8}(x - (-3))$
$y = \frac{3}{8}x + \frac{9}{8} + 1$
$y = \frac{3}{8}x + \frac{17}{8}$

49. $(15, -10), (18, 12)$
$m = \frac{12-(-10)}{18-15} = \frac{22}{3}$
$y - (-10) = \frac{22}{3}(x - 15)$
$y + 10 = \frac{22}{3}x - 110$
$y = \frac{22}{3}x - 120$

53. $(4, \frac{3}{8}), (12, -\frac{1}{4})$
$m = \frac{-\frac{1}{4}-\frac{3}{8}}{12-4} = \frac{-\frac{5}{8}}{8} = -\frac{5}{8} \bullet \frac{1}{8} = -\frac{5}{64}$
$y - (-\frac{1}{4}) = -\frac{5}{64}(x - 12)$
$y + \frac{1}{4} = -\frac{5}{64}x + \frac{15}{16}$
$y = -\frac{5}{64}x + \frac{11}{16}$

57. $(m, m + 2n), (n, m - 2n)$, $m \neq n$
slope $= \frac{(m-2n)-(m+2n)}{n-m} = \frac{4n}{m-n}$
$y - (m + 2n) = \frac{4n}{m-n}(x - m)$
$y - m - 2n = \frac{4n}{m-n}x - \frac{4mn}{m-n}$
$y = \frac{4n}{m-n}x - \frac{4mn}{m-n} + m + 2n,$
$y = \frac{4n}{m-n}x - \frac{4mn}{m-n} + \frac{(m+2n)(m-n)}{m-n}$
$y = \frac{4n}{m-n}x + \frac{m^2-3mn-2n^2}{m-n}$

61. $y - (-3) = -2(x - \frac{1}{2})$
$y + 3 = -2x + 1$
$y = -2x - 2$

65. $5y - 3x = 4$
$y = \frac{3}{5}x + \frac{4}{5}$ Solve for y; $m = \frac{3}{5}$.
Use $m = \frac{3}{5}$ Parallel lines have the same slope.
$y - 2 = \frac{3}{5}(x - (-5))$ Use the point (–5, 2).
$y = \frac{3}{5}x + 5$

69. Point is (–3, 0), $m = 5$.
$y - y_1 = m(x - x_1)$
$y - 0 = 5(x - (-3))$
$y = 5x + 15$

73. Undefined slope means a vertical line, which is of the form $x = k$. Thus we want the line $x = -1$.

77. $x = 2$

81. First find the wcf for a –11.5° temperature for 15 mph and 20 mph winds.
At 15 mph we use the (temperature, wcf) points (–10, –45) and (–15, –51). We compute y in the ordered pair (–11.5, y). The wcf is –46.8°.
At 20 mph we use the (temperature, wcf) points (–10, –52) and (–15, –60). Compute y in the ordered pair (–11.5, y) and obtain the wcf –54.4°.
Now we have the ordered pairs (mph, wcf) of (15, –46.8°) and (20, –54.4°). We use these to compute y in the ordered pair (18.5, y). The value of y is –52.12, so the required wind chill factor for –11.5° and 18.5 mph is –52.1°.

85. The line which contains the points A and C (–5, 8) and (–8, 4) can be computed to be $y = \frac{4}{3}x + 14\frac{2}{3}$. Thus the line through B and C must have slope $-\frac{3}{4}$ and pass through C; using this slope and the point C we obtain the equation $y = -\frac{3}{4}x - 2$. Now, let (x, y) be the coordinates of B. Then if its distance to C is 12, then
$12 = \sqrt{(x-(-8))^2 + (y-4)^2}$
$144 = x^2 + 16x + 64 + y^2 - 8y + 16$
$x^2 + 16x + y^2 - 8y = 64$, and since $y = -\frac{3}{4}x - 2$, we have
$x^2 + 16x + (-\frac{3}{4}x - 2)^2 - 8(-\frac{3}{4}x - 2) = 64,$
which can be transformed into
$25x^2 + 400x - 704 = 0$
$(5x - 8)(5x + 88) = 0$
$x = 1\frac{3}{5}$ or $-17\frac{3}{5}$. From the figure it is clear that we want the value $x = 1\frac{3}{5}$.
(The other value would give a solution for B also, but not corresponding to the figure.) Since $y = -\frac{3}{4}x - 2$,
$y = -\frac{3}{4}\bullet\frac{8}{5} - 2 = -3\frac{1}{5}$. Thus the point B is $(1\frac{3}{5}, -3\frac{1}{5})$.

89. Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two different points on the line $y = \frac{1}{3}x + 2$.
Then $y_1 = \frac{1}{3}x_1 + 2$, and
$y_2 = \frac{1}{3}x_2 + 2$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(\frac{1}{3}x_2 + 2) - (\frac{1}{3}x_1 + 2)}{x_2 - x_1}$$
$$= \frac{\frac{1}{3}x_2 - \frac{1}{3}x_1}{x_2 - x_1} = \frac{\frac{1}{3}(x_2 - x_1)}{x_2 - x_1} = \frac{1}{3}.$$

93. $\begin{cases} 2x + y = 4 \\ x - y = 6 \end{cases}$

$\begin{cases} 2x + y = 4 \\ y = x - 6 \end{cases}$ Solve second equation for y.

$2x + (x - 6) = 4$ Replace y by $x - 6$ in first equation.

$x = \frac{10}{3}$ Solve for x.

$y = x - 6$ Second equation.

$y = \frac{10}{3} - 6$ Replace x by $\frac{10}{3}$.

$= -\frac{8}{3}$ Solve for y.

The point is $(3\frac{1}{3}, -2\frac{2}{3})$.

97. $\begin{cases} \frac{1}{2}x - 3y = 1 \\ \frac{2}{3}y = x + 4 \end{cases}$

$\begin{cases} x = 6y + 2 \\ 2y = 3x + 12 \end{cases}$ Solve the first equation for x.

$2y = 3(6y + 2) + 12$ Replace x in second equation by $6y + 2$.

$y = -\frac{9}{8}$ Solve for y.

$x = 6y + 2$

$x = 6(-\frac{9}{8}) + 2 = -\frac{19}{4}$ Replace y in first equation by $-\frac{9}{8}$.

The point is $(-4\frac{3}{4}, -1\frac{1}{8})$.

101. $-A + B = 10$ $\quad$ $A + B = 4$
$B = 4 - A$
$-A + (4 - A) = 10$
$-6 = 2A$
$-3 = A$ $\quad$ $B = 4 - (-3) = 7$
$(-3, 7)$

105. $y = x^2 + 24$ $\quad$ $y = 10x + 3$
$10x + 3 = x^2 + 24$
$0 = x^2 - 10x + 21$
$0 = (x - 3)(x - 7)$
$x = 3, 7$ $\quad$ $y = 10(3) + 3 = 33$
$y = 10(7) + 3 = 73$
$(3, 33), (7, 73)$

109. $y = x^2 - 3x - 5$ $\quad$ $y = 2x + 1$
$2x + 1 = x^2 - 3x - 5$
$0 = x^2 - 5x - 6$
$0 = (x - 6)(x + 1)$
$x = -1, 6$ $\quad$ $y = 2(-1) + 1 = -1$
$y = 2(6) + 1 = 13$
$(-1, -1), (6, 13)$

113. $y = x^2 + 3x - 70$ $\quad$ $y = x^2 - x - 2$
$x^2 - x - 2 = x^2 + 3x - 70$
$68 = 4x$
$17 = x$ $\quad$ $y = 17^2 - 17 - 2 = 270$
$(17, 270)$

Exercise 3–3

1. A function is a relation in which no first element repeats.

5. Not a function; the first element 2 repeats. dmn {-10,2,4}
rng {-5,9,12,13}

9. { (1, 1), (8, 2), (27, 3), (−1, −1), (−8, −2), (−27, −3) }
A function, not one to one.
Domain: { ±1, ± 8, ± 27 }
Range: { ±1, ± 2, ± 3 }

13. $g(x) = \sqrt{2x - 1}$; the domain is $2x - 1 \geq 0$, $2x \geq 1$, $x \geq \frac{1}{2}$. Thus $D = \{ x \mid x \geq \frac{1}{2} \}$. $g(-4)$, $g(0)$ are not defined since −4, 0, are not in the domain of g.

$g(\frac{1}{2}) = \sqrt{2(\frac{1}{2}) - 1} = 0$

$g(7) = \sqrt{2(7) - 1} = \sqrt{13}$

$g(3\sqrt{2}) = \sqrt{2(3\sqrt{2}) - 1} = \sqrt{6\sqrt{2} - 1}$

$g(c - 1) = \sqrt{2(c - 1) - 1} = \sqrt{2c - 3}$

> Think of $f(x)$ notation as a template. In the following problem the x is just a space holder. We could as easily use any other symbol, such as a box to be filled in.
> $m(x) = 3x^2 - x - 11$
> $m(\text{o}) = 3 \cdot (\square)^2 - (\square) - 11$
> Whatever you want "m of" is filled in for the $\square$ symbol.

17. $m(x) = 3x^2 - x - 11$; the domain is all real numbers since there are no restrictions on the operations of multiplication and subtraction. $D = R$.
$m(-4) = 3(-4)^2 - (-4) - 11 = 41$
$m(0) = 3(0)^2 - (0) - 11 = -11$
$m(\frac{1}{2}) = 3(\frac{1}{2})^2 - (\frac{1}{2}) - 11 = -\frac{43}{4}$
$m(7) = 3(7)^2 - (7) - 11 = 129$
$m(3\sqrt{2}) = 3(3\sqrt{2})^2 - (3\sqrt{2}) - 11$
$= 3(9 \cdot 2) - 3\sqrt{2} - 11$
$= 43 - 3\sqrt{2}$
$m(c - 1) = 3(c - 1)^2 - (c - 1) - 11$
$= 3c^2 - 7c - 7$

21. $h(x) = x^3 - 4$; Domain is R.
$h(-4) = (-4)^3 - 4 = -68$ $\quad$ $h(0) = 0^3 - 4 = -4$
$h(\frac{1}{2}) = (\frac{1}{2})^3 - 4 = \frac{1}{8} - 4 = -3\frac{7}{8}$ $\quad$ $h(7) = 7^3 - 4 = 339$
$h(3\sqrt{2}) = (3\sqrt{2})^3 - 4 = 3^3\sqrt{2^3} - 4 = 27(2\sqrt{2}) - 4 = 54\sqrt{2} - 4$
$h(c - 1) = (c - 1)^3 - 4 = (c^3 - 3c^2 + 3c - 1) - 4$
$= c^3 - 3c^2 + 3c - 5$

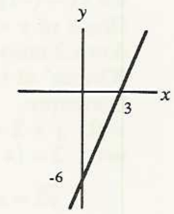
25. $f(x + h) = (x + h)^2 - 3(x + h) - 5$
$= x^2 + 2hx + h^2 - 3x - 3h - 5$, so the quotient is
$$\frac{(x^2 + 2hx + h^2 - 3x - 3h - 5) - (x^2 - 3x - 5)}{h}$$
$$= \frac{2hx + h^2 - 3h}{h} = \frac{h(2x + h - 3)}{h} = 2x - 3 + h.$$

29. (a) $g(1) = 1$
$f(g(1)) = f(1) = -3$
(b) $g(3) = \frac{3}{2}$
$f(g(3)) = f(\frac{3}{2}) = 2(\frac{3}{2}) - 5 = -2$
(c) $f(0) = -5$
$g(f(0)) = g(-5) = \frac{2(-5)}{-5 + 1} = \frac{5}{2}$
Work from the inside out. Compute $f(0)$ first, then plug this value into $g(x)$.
(d) $f(\frac{1}{2}) = -4$
$g(f(\frac{1}{2})) = g(-4) = \frac{2(-4)}{-4 + 1} = \frac{8}{3}$

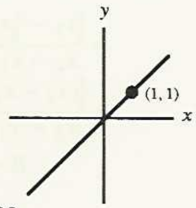
33. $\frac{f(1) - g(2)}{f(1) + g(2)}$
$\frac{(5(1) - 1) - (2(2) + 2)}{(5(1) - 1) - (2(2) + 2)} = \frac{4 - 6}{4 + 6} = -\frac{1}{5}$

37. $f(x) + 3$
$(5x - 1) + 3$
$5x + 2$

41. $f(x) = 2x - 6$
$y = 2x - 6$
Intercepts: (0,−6), (3,0)



45. $h(x) = x$
$y = x$
Intercepts: (0,0)
Another point is (1,1).



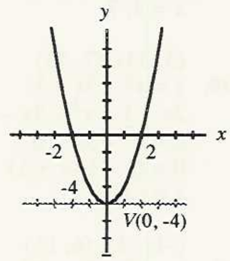
49. $C(m) = 0.34m + 500$

53. We have two points (h, v): (0, 8) and (340, 25). We assume a linear function $v = mh + b$ and need to find m and b. We know that $8 = m(0) + b$, so $b = 8$, so we know $v = mh + 8$. Using the second point we find $25 = m(340) + 8$, $17 = 340m$, $\frac{17}{340} = \frac{1}{20} = m$. Thus the relation $v = \frac{1}{20}h + 8$ describes velocity as a function of height.

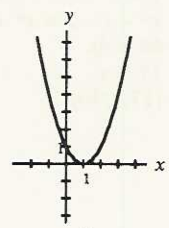
57. $A = 2\pi r^2 + 2\pi rh$, and if $r + h = 20$ then $r = 20 - h$, so
$A = 2\pi(20 - h)^2 + 2\pi(20 - h)h$,
$A = 2\pi(400 - 40h + h^2) + 40\pi h - 2\pi h^2$,
$A = 800\pi - 80\pi h + 2\pi h^2 + 40\pi h - 2\pi h^2$, $A = 800\pi - 40\pi h$

Exercise 3–4

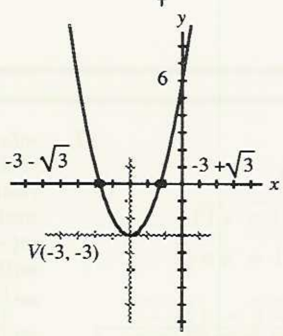
1. $y = x^2 - 4$
Graph $y = x^2$ shifted down 4 units.
Vertex at (0, –4)
Intercepts:
x=0: $y = -4$
y=0: $0 = x^2 - 4$
$x = \pm 2$



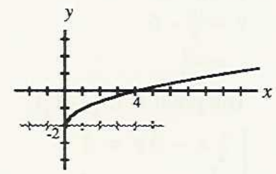
5. $y = (x - 1)^2$
Graph $y = x^2$ shifted right 1 unit.
Vertex at (1, 0)
Intercepts:
x=0: $y = (-1)^2 = 1$
y=0: $0 = (x - 1)^2$
$0 = x - 1$
$1 = x$



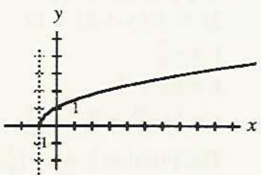
9. $y = (x + 3)^2 - 3$
$y = (x - (-3))^2 - 3$
Graph of $y = x^2$ shifted down 3 units and left 3 units.
Vertex at (–3, –3).
Intercepts:
x=0: $y + 3 = (0 + 3)^2$
$y = 6$
y=0: $3 = (x + 3)^2$
$\pm\sqrt{3} = x + 3$
$-3 \pm\sqrt{3} = x$



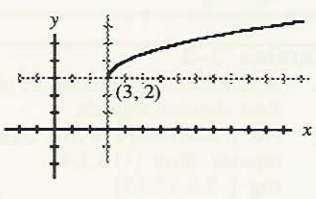
13. $y = \sqrt{x} - 2$
Graph $y = \sqrt{x}$ shifted down 2 units.
Vertex at (0, –2)
Intercepts:
x=0: $y = 0 - 2 = -2$
y=0: $0 = \sqrt{x} - 2$
$2 = \sqrt{x}$
$4 = x$



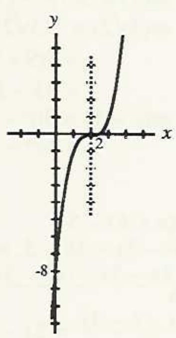
17. $y = \sqrt{x + 1}$
$y = \sqrt{x - (-1)}$
Rewrite in terms of $x - c$.
Graph $y = \sqrt{x}$ shifted left 1 unit.
Vertex at (–1, 0)
Intercepts:
x=0: $y = \sqrt{0 + 1} = 1$
y=0: $0 = \sqrt{x + 1}$
$0 = x + 1$
$-1 = x$



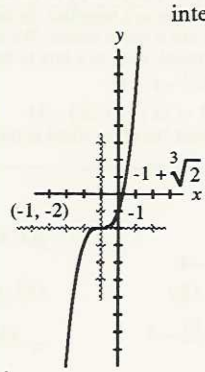
21. $y = \sqrt{x - 3} + 2$
Graph $y = \sqrt{x}$ shifted left 3 units, up 2 units. Vertex at (3, 2).
Intercepts:
x=0: $y = \sqrt{-3} + 2$, not real so no y-intercept.
y=0: $0 = \sqrt{x - 3} + 2$
$-2 = \sqrt{x - 3}$; A square root is non–negative, so no x-intercept.



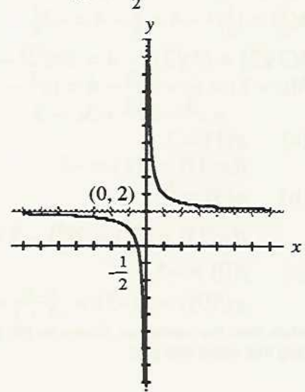
25. $y = (x - 2)^3$
Graph $y = x^3$ shifted right 2 units.
"Origin" at (2, 0)
Intercepts:
x=0: $y = (-3)^2$
$= -8$
y=0:
$0 = (x - 2)^3$
$0 = x - 2$
$2 = x$



29. $y = (x + 1)^3 - 2$
$y = (x - (-1))^3 - 2$
Graph of $y = x^3$ shifted left 1 unit, down 2 units.
"Origin" at (–1, –2).
Intercepts:
x=0: $y + 2 = 1^3$, $y = -1$
y=0: $2 = (x + 1)^3$
$\sqrt[3]{2} = x + 1$
$-1 + \sqrt[3]{2} = x \approx 0.3$

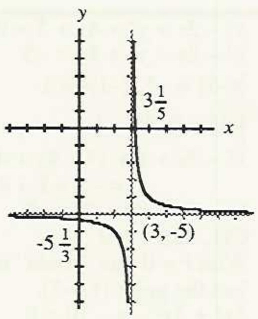


33. $y = \dfrac{1}{x} + 2$
Graph of $y = \frac{1}{x}$ shifted 2 units up; "origin" at (0, 2).
Intercepts:
x=0: $\frac{1}{0}$ is undefined, so no y-intercept.
y=0: $0 = \frac{1}{x} + 2$
$-2 = \frac{1}{x}$
$-2x = 1$
$x = -\frac{1}{2}$

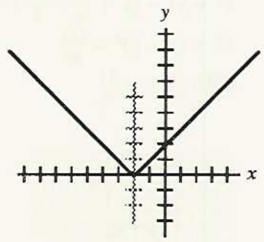


37. $y = \dfrac{1}{x-3} - 5$

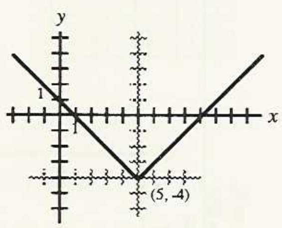
Graph of $y = \frac{1}{x}$ shifted 3 units right, 5 units down;
"origin" at (3, –5)
Intercepts:
x=0: $y = -\frac{1}{3} - 5$
$y = -5\frac{1}{3}$
y=0: $5 = \frac{1}{x-3}$
$5(x-3) = 1$
$x - 3 = \frac{1}{5}$
$x = 3\frac{1}{5}$



41. $y = |x + 2|$
$y = |x - (-2)|$
Rewrite in terms of $x - c$.
Graph of $y = |x|$, shifted left 2 units.
"origin" (–2, 0)
Intercepts:
x=0: $y = |2| = 2$
y=0: $0 = |x + 2|$
$0 = x + 2$
$-2 = x$

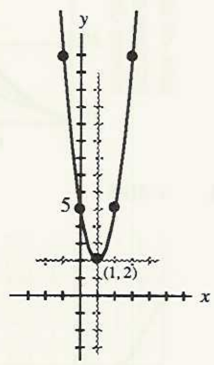


45. $y = |x - 5| - 4$
Graph of $y = |x|$, shifted right 5 units and down 4 units.
"origin" (5, –4)
Intercepts:
x=0: $y = |-5| - 4$
$= 1$
y=0: $4 = |x - 5|$
$x - 5 = 4$ or $x - 5 = -4$
$x = 9$ or $x = 1$

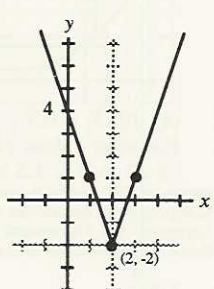


49. $y = 3(x-1)^2 + 2$
Graph of $y = x^2$, shifted up 2 units, right 1 unit, vertically scaled 3 units.
"Origin" at (1, 2).
Intercepts:
x=0: $y - 2 = 3(-1)^2$
$y = 5$
y=0: $-2 = 3(x-1)^2$
$-\frac{2}{3} = (x-1)^2$; no real solutions so no x-intercepts.
Additional points:

x:	–1	2	3
y:	14	5	14



53. $y = |3x - 6| - 2$
$y = 3|x - 2| - 2$
Graph of $y = |x|$, shifted down 2 units, right 2 units, vertically scaled 3 units. "Origin" at (2, –2)
Intercepts:
x=0: $y = |-6| - 2 = 4$
y=0: $2 = 3|x - 2|$
$\frac{2}{3} = |x - 2|$, so
$x - 2 = \frac{2}{3}$ or $x - 2 = -\frac{2}{3}$
$x = 2\frac{2}{3}$ or $x = 1\frac{1}{3}$



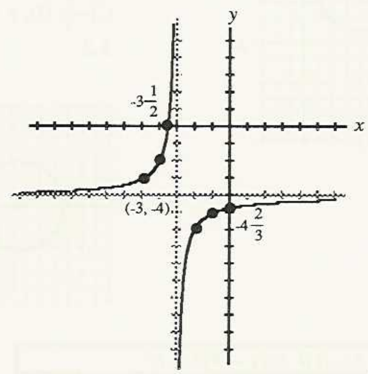
Additional points:

x:	1	3
y:	1	1

57. $y = \dfrac{-2}{x+3} - 4$
$y = \dfrac{-2}{x-(-3)} - 4$
Graph of $y = \frac{1}{x}$, shifted down 4 units, left 3 units, vertically scaled –2 units. "Origin" at (–3, –4).
Intercepts:
x=0: $y = -\frac{2}{3} - 4 = -4\frac{2}{3}$
y=0: $0 = \frac{-2}{x+3} - 4$
$4 = \frac{-2}{x+3}$
$4x + 12 = -2$
$4x = -14$
$x = -3\frac{1}{2}$

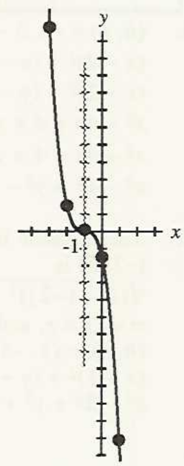
Additional points:

x:	–5	–4	–2	–1
y:	–3	–2	–6	–5

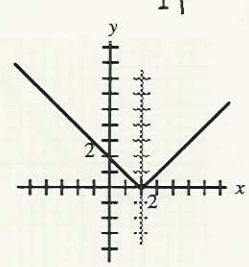


61. $y = -\frac{3}{2}(x+1)^3$
$y = -\frac{3}{2}(x - (-1))^3$
Graph of $y = x^3$ shifted left 1 unit, vertically scaled $-1\frac{1}{2}$ units. "Origin" at (–1, 0)
Intercepts:
x=0: $2y = -3$
$y = -1\frac{1}{2}$
y=0: $0 = -3(x+1)^3$
$0 = (x+1)^3$
$0 = x + 1$
$-1 = x$
Additional points:

x:	–3	–2	1
y:	12	$1\frac{1}{2}$	–12

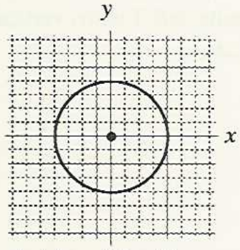


65. $y = |x - 2|$
Graph $y = |x|$ shifted right 2 units.
Vertex at (2, 0)
Intercepts:
x=0: $y = |-2| = 2$
y=0: $0 = |x - 2|$
$0 = x - 2$
$2 = x$

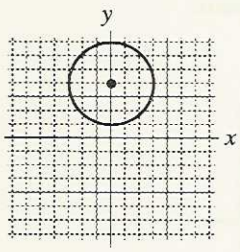


Exercise 3–5

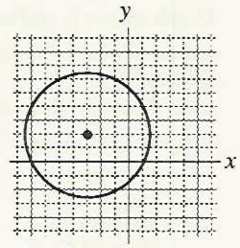
1. $x^2 + y^2 = 16$
   $C(0, 0),\ r = \sqrt{16} = 4$



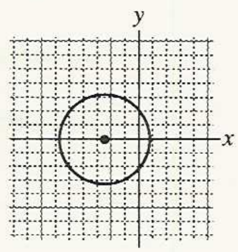
5. $x^2 + (y - 4)^2 = 9$
   $C(0, 4),\ r = 3$



9. $(x + 3)^2 + (y - 2)^2 = 20$
   $C(-3, 2),\ r = \sqrt{20} = 2\sqrt{5}$
   ≈ 4.47

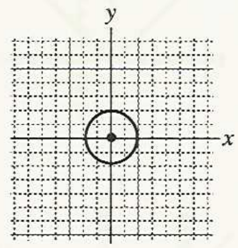


13. $x^2 + 5x + y^2 = 4$
    $\frac{1}{2}(5) = \frac{5}{2},\ (\frac{5}{2})^2 = \frac{25}{5},$
    $x^2 + 5x + \frac{25}{4} + y^2 = 4 + \frac{25}{4}$
    $(x + \frac{5}{2})^2 + y^2 = \frac{41}{4}$
    $C(-\frac{5}{2}, 0),\ r = \sqrt{\frac{41}{4}} = \frac{\sqrt{41}}{2} \approx$
    3.2

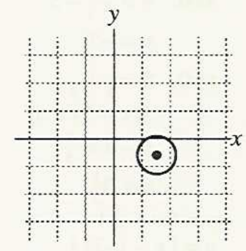


17. $x^2 - 2x + y^2 + 4y + 5 = 0$
    $x^2 - 2x + y^2 + 4y = -5$
    $\frac{1}{2}(-2) = -1,\ (-1)^2 = 1$
    $\frac{1}{2}(4) = 2,\ 2^2 = 4,$
    $x^2 - 2x + 1 + y^2 + 4y + 4$
    $= -5 + 1 + 4$
    $(x - 1)^2 + (y + 2)^2 = 0$
    $C(1, -2),\ r = 0$
    With $r = 0$ this "circle" is just the point $(1, -2)$.

21. $3x^2 + 3y^2 - y - 10 = 0$
    $x^2 + y^2 - \frac{1}{3}y = \frac{10}{3}$
    $\frac{1}{2}(-\frac{1}{3}) = -\frac{1}{6},\ (-\frac{1}{6})^2 = \frac{1}{36},$
    $x^2 + y^2 - \frac{1}{3}y + \frac{1}{36} = \frac{10}{3} + \frac{1}{36}$
    $x^2 + (y - \frac{1}{6})^2 = \frac{121}{36}$
    $C(0, \frac{1}{6}),\ r = \frac{11}{6}$



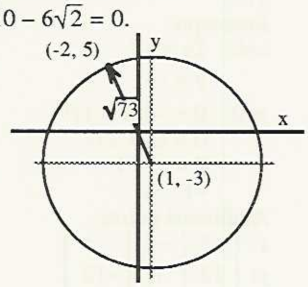
25. $(2x - 3)^2 + (2y + 1)^2 = 2$
    $[2(x - \frac{3}{2})]^2 + [2(y + \frac{1}{2})]^2 = 2$
    $4(x - \frac{3}{2})^2 + 4(y + \frac{1}{2})^2 = 2$
    $(x - \frac{3}{2})^2 + (y + \frac{1}{2})^2 = \frac{1}{2}$
    $C(\frac{3}{2}, -\frac{1}{2}),\ r = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \approx 0.7$



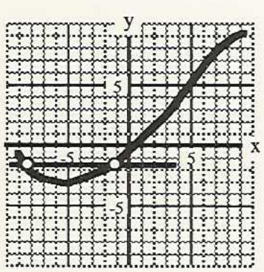
$$\boxed{(x - h)^2 + (y - k)^2 = r^2}$$

29. $(h, k) = (2, 3 - \sqrt{2}),\ r = \sqrt{5}$
    $(x - 2)^2 + (y - (3 - \sqrt{2}))^2 = (\sqrt{5})^2$
    $(x - 2)^2 + (y - (3 - \sqrt{2}))^2 = 5$
    $x^2 - 4x + 4 + y^2 - 2(3 - \sqrt{2})y + (3 - \sqrt{2})^2 = 5$
    $x^2 - 4x + 4 + y^2 - 2(3 - \sqrt{2})y + 9 - 6\sqrt{2} + 2 = 5$
    $x^2 - 4x + y^2 - (6 - 2\sqrt{2})y + 10 - 6\sqrt{2} = 0.$

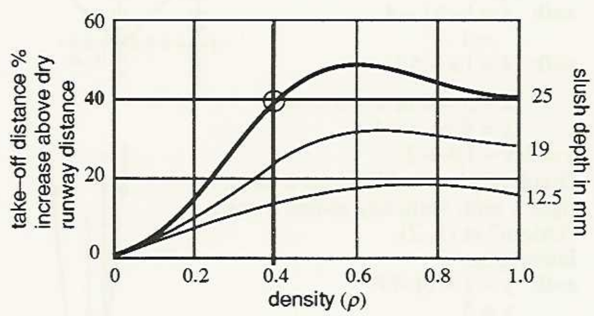
33. The distance from $(1, -3)$ to $(-2, 5)$ is
    $\sqrt{(1 - (-2))^2 + (-3 - 5)^2}$
    $= \sqrt{73} = r$, and
    $(h, k) = (1, -3).$
    $(x - 1)^2 + (y + 3)^2 = 73$
    $x^2 - 2x + y^2 + 6y - 63 = 0$



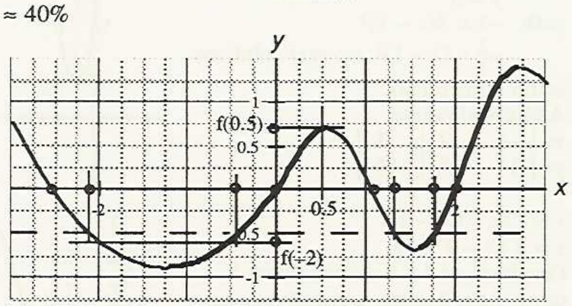
37.



Function (passes vertical line test); not one-to-one (fails horizontal line test).



41. $\approx 40\%$



45. (a) $f(0.5) = 0.7$; (b) $f(-2) = -0.6$

49. For what values of x is f increasing?
    -1.25 to 0.5, 1.5 to 2.7.

To determine if a function is even or odd,
(a) Compute $f(-x)$. Remember that this only changes terms with odd exponents on x.
(b) Compute $-f(x)$. This means change the sign of every term in the original expression.
(c) See if the results of (a) or (b) are the same as the original function.

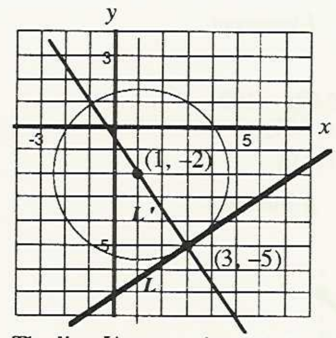
53. $y = \dfrac{4}{x}$

$$f(-x) = \frac{4}{-x} = -\frac{4}{x} = -f(x)$$; odd, origin symmetry

57. $h(x) = x^3$
$h(-x) = (-x)^3$
$= -x^3 = -h(x)$; odd, origin symmetry

61. $g(x) = \dfrac{x}{x^3 - 1}$
$g(-x) = \dfrac{-x}{(-x)^3 - 1} = \dfrac{-x}{-x^3 - 1} = \dfrac{x}{x^3 + 1}$
$-g(x) = \dfrac{-x}{x^3 - 1}$
Neither even nor odd since $g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$.

65. $f(x) = \sqrt{4 - x^2}$
$f(-x) = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2} = f(x)$;
even, y-axis symmetry

69.


The line L' passes through the points (1, –2) and (3, –5). The equation of L' can be found (section 3–2) to be
[L'] $y = -\frac{3}{2}x - \frac{1}{2}$ $(m = -\frac{3}{2})$.
Thus the slope of L is $\frac{2}{3}$ (L and L' are perpendicular (section 3–2)). Using (3, –5) with $m = \frac{2}{3}$ we find the equation of L to be $y = \frac{2}{3}x - 7$.

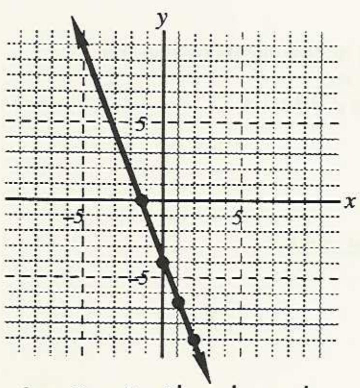
Chapter 3 Review

The three points you use may differ from those below, but the x- and y-intercepts, and the graph, should be the same.

1. $2y = -5x - 8$
$y = \dfrac{-5x - 8}{2}$

x	y
0	–4
1	$-\frac{13}{2}$
2	–9

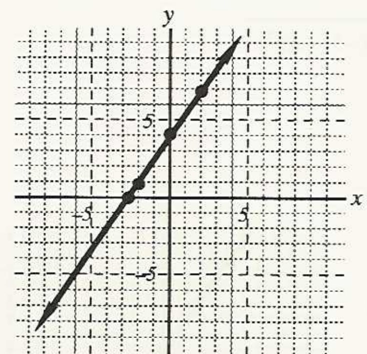
Intercepts: x=0: (0, –4)
y=0: $2(0) = -5x - 8$
$x = -\frac{8}{5}$ $(-1\frac{3}{8}, 0)$



3. $3x - 2y + 8 = 0$
$3x + 8 = 2y$
$y = \dfrac{3x + 8}{2}$

x	y	
–2	1	
0	4	(y-intercept)
2	7	

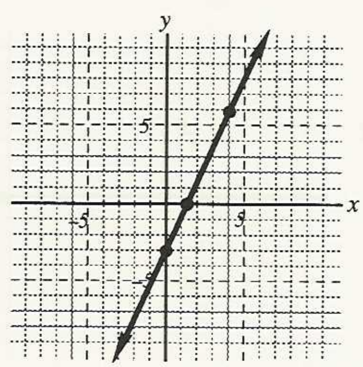
Intercepts: y=0: $3x - 2(0) + 8 = 0$
$x = -\frac{8}{3}$ $(-2\frac{2}{3}, 0)$



5. $\frac{3}{4}x - \frac{1}{3}y = 1$
$\frac{3}{4}x - 1 = \frac{1}{3}y$
$\frac{9}{4}x - 3 = y$

x	y	
–4	–12	
0	–3	(y-intercept)
4	6	

Intercepts: y=0: $\frac{3}{4}x - \frac{1}{3}(0) = 1$; $x = \frac{4}{3}$ $(1\frac{1}{3}, 0)$



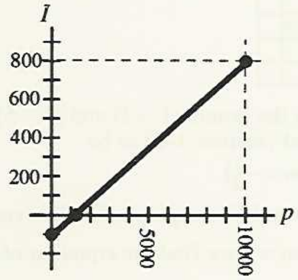
7.

p	I	
0	-100	I-intercept
10000	800	

p-intercept: $I=0$

$0 = 0.09p - 100$
$100 = 0.09p$
$p = 1111\frac{1}{9}$

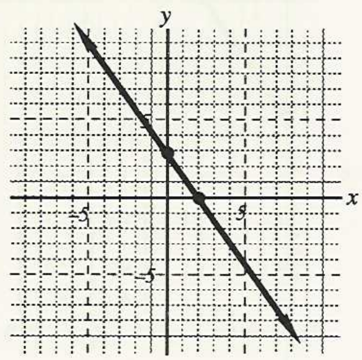
$1111\frac{1}{9}$	0	p-Intercept



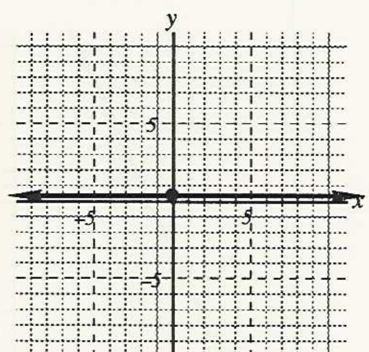
9. $(-2, \sqrt{8}), (-6, \sqrt{2})$
$\left(\frac{-2 + (-6)}{2}, \frac{2\sqrt{2} + \sqrt{2}}{2}\right) = (-4, \frac{3}{2}\sqrt{2})$

11. $(\sqrt{8}, 2), (\sqrt{2}, -3)$
$\sqrt{(\sqrt{2} - 2\sqrt{2})^2 + (-3 - 2)^2}$
$\sqrt{(-\sqrt{2})^2 + (-5)^2}$
$\sqrt{2 + 25}$
$\sqrt{27}$
$3\sqrt{3}$

13. $2y = 6 - 3x$
x-intercept (y=0):
$0 = 6 - 3x \,;\, x = 2$
y-intercept (x=0):
$2y = 6 \,;\, y = 3$
$m = -1\frac{1}{2}$



15. $y = \frac{4}{9}$; This is a horizontal line in which each value of y is $\frac{4}{9}$. $m = 0$



17. $P_1 = (-3, -2\frac{1}{3}), P_2 = (5, \frac{2}{3})$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3} - (-2\frac{1}{3})}{5 - (-3)} = \frac{3}{8}$

19. $(\sqrt{3}, -1), (2\sqrt{3}, 4)$
$\frac{4 - (-1)}{2\sqrt{3} - \sqrt{3}} = \frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

21. $(2, -3), (3, 1)\,;\, m = \frac{1 - (-3)}{3 - 2} = 4$
$y - y_1 = m(x - x_1)$
$y - (-3) = 4(x - 2)$
$y + 3 = 4x - 8$
$y = 4x - 11$

23. A line with slope -6 which passes through the point $(\frac{1}{2}, -4)$.
$y - y_1 = m(x - x_1)$
$y - (-4) = -6(x - \frac{1}{2})$
$y + 4 = -6x + 3$
$y = -6x - 1$

25. A line which is perpendicular to the line $2y + 5x = 8$ and passes through the point $(4, -\frac{1}{5})$.
$2y + 5x = 8$
$2y = -5x + 8$
$y = -\frac{5}{2}x + 4\,;\; m = -\frac{5}{2}$, so use $+\frac{2}{5}$ for slope (to have a line perpendicular to the first).
$y - y_1 = m(x - x_1)$
$y - (-\frac{1}{5}) = \frac{2}{5}(x - 4)$
$y + \frac{1}{5} = \frac{2}{5}x - \frac{8}{5}$
$y = \frac{2}{5}x - \frac{9}{5}$

27. $y = \boxed{3x + 2}$

$-\boxed{y} + x = -5$
$-(3x + 2) + x = -5$
$-2x = -3$
$x = \frac{3}{2}$

$y = 3 \cdot \frac{3}{2} + 2$
$y = \frac{9}{2} + \frac{4}{2}$
$y = \frac{13}{2}$
$(1\frac{1}{2}, 6\frac{1}{2})$

29. Let (a, b) and (c, d) be two points on the line $y = 3x - 4$ so that $a \neq c$.
Then $b = 3a - 4$ and $d = 3c - 4$.
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{d - b}{c - a}$ if $c \neq a$.
$m = \frac{(3c - 4) - (3a - 4)}{c - a}$
$m = \frac{3c - 3a}{c - a}$
$m = \frac{3(c - a)}{c - a}$
$m = 3$

31. It is not a function since it is not true that all the first elements are different. dmn {–3,1,4}, rng {2,4,5,8}

33. Function since all first elements are different. One-to-one since all the second elements are different. dmn {–10,2,3,4} rng {–5,2,12,13}

35. $\{ (x, y) \mid x + 3y = 6, x \varepsilon \{-3, 9, \sqrt{18}, \frac{3}{4}, \pi \}$ (Note that $\sqrt{18} = 3\sqrt{2}$.)

$x + 3y = 6$, so $3y = -x + 6$, so $y = \dfrac{-x+6}{3}$

$\{ (-3, \frac{-(-3)+6}{3}), (9, \frac{-9+6}{3}), (3\sqrt{2}, \frac{-3\sqrt{2}+6}{3}), (\frac{3}{4}, \frac{-\frac{3}{4}+6}{3}), (\pi, \frac{-\pi+6}{3}) \}$

$\{(-3, 3), (9, -1), (3\sqrt{2}, -\sqrt{2}+2), (\frac{3}{4}, \frac{7}{4}), (\pi, -\frac{\pi}{3}+2)\}$; one to one

37. $f(x) = \dfrac{1-x^2}{4x-3}$

Implied domain: $4x - 3 \neq 0$, so $x \neq \frac{3}{4}$ is the implied domain.

$f(-4) = \dfrac{1-(-4)^2}{4(-4)-3} = \dfrac{15}{19}$

$f(0) = \dfrac{1-0^2}{4(0)-3} = -\dfrac{1}{3}$

$f(\frac{1}{2}) = \dfrac{1-(\frac{1}{2})^2}{4(\frac{1}{2})-3} = -\dfrac{3}{4}$

$f(3\sqrt{5}) = \dfrac{1-(3\sqrt{5})^2}{4(3\sqrt{5})-3} = \dfrac{1-45}{12\sqrt{5}-3}$

$= \dfrac{-44}{12\sqrt{5}-3}$

$f(c-2) = \dfrac{1-(c-2)^2}{4(c-2)-3} = \dfrac{-c^2+4c-3}{4c-11}$

39. $v(x) = 3 - 2x - x^2$

Implied domain: R, since there are no radicals or fractions to restrict values of x.

$v(-4) = 3 - 2(-4) - (-4)^2 = -5$

$v(0) = 3 - 2(0) - (0)^2 = 3$

$v(\frac{1}{2}) = 3 - 2(\frac{1}{2}) - (\frac{1}{2})^2 = 1\frac{3}{4}$

$v(3\sqrt{5}) = 3 - 2(3\sqrt{5}) - (3\sqrt{5})^2$

$= 3 - 6\sqrt{5} - 9(5) = -42 - 6\sqrt{5}$

$v(c-2) = 3 - 2(c-2) - (c-2)^2 = -c^2 + 2c + 3$

41. $\dfrac{[f(x+h)] - [f(x)]}{h}$

$= \dfrac{[(x+h)^2 - 5(x+h) - 5] - [x^2 - 5x - 5]}{h}$

$= \dfrac{[x^2 + 2hx + h^2 - 5x - 5h - 5] - [x^2 - 5x - 5]}{h}$

$= \dfrac{2hx - 5h + h^2}{h} = \dfrac{h(2x - 5 + h)}{h}$

$= 2x - 5 + h$

43. If $f(x) = 2x + 3$ and $g(x) = 1 - 4x$, compute

(a) $g(-3) = 13$

$f(g(-3)) = f(13) = 29$

(b) $g(\frac{1}{2}) = -1$

$f(g(\frac{1}{2})) = f(-1) = 1$

(c) $f(\frac{a}{a+b}) = \dfrac{2a}{a+b} + 3$

$= \dfrac{2a}{a+b} + \dfrac{3(a+b)}{a+b} = \dfrac{5a+3b}{a+b}$

$g(f(\frac{a}{a+b})) = g(\frac{5a+3b}{a+b}) = 1 - 4(\frac{5a+3b}{a+b})$

$= \dfrac{a+b}{a+b} - \dfrac{4(5a+3b)}{a+b}$

$= \dfrac{-19a - 11b}{a+b}$

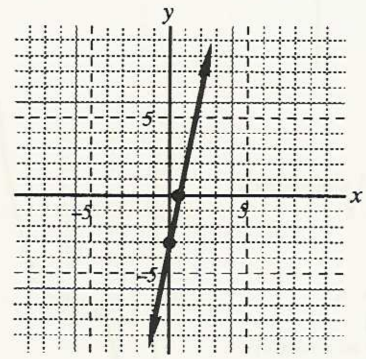
45. Graph the linear function $f(x) = 5x - 3$

$y = 5x - 3$

x-intercept (y=0): $0 = 5x - 3$

$\frac{3}{5} = x$

y-intercept (x=0): $y = -3$



47. $f(x) = (x - 3\frac{1}{2})^2 + 5$

This is the graph of $y = x^2$ shifted to a vertex of $(3\frac{1}{2}, 5)$.

x-intercept (y=0):

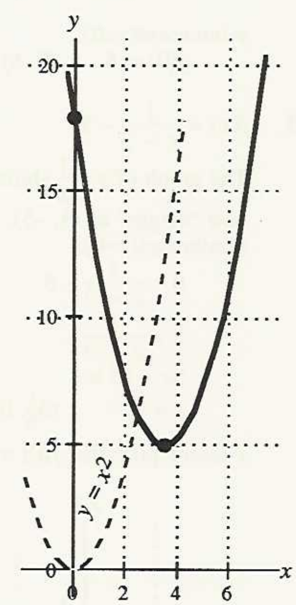
$0 = (x - 3\frac{1}{2})^2 + 5$

$-5 = (x - 3\frac{1}{2})^2$ No solution since the left member is negative and the right member is nonnegative.

y-intercept (x=0):

$f(0) = (-3\frac{1}{2})^2 + 5$

(0, 17.25)



49. $f(x) = \sqrt{x + \frac{5}{2}} - 5$

The graph of $y = \sqrt{x}$, with "origin" shifted to $(-2\frac{1}{2}, -5)$.

x-intercept(y=0): $0 = \sqrt{x + 2.5} - 5$

$5 = \sqrt{x + 2.5}$

$25 = x + 2.5$

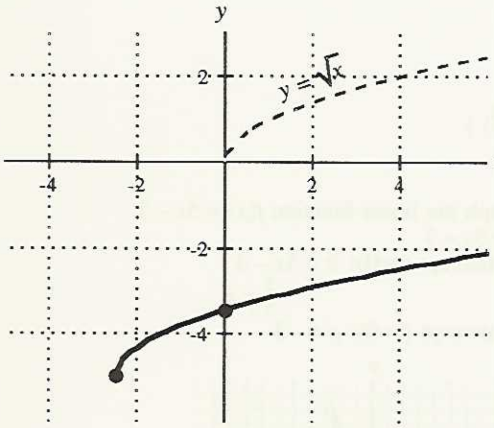
$21.5 = x$

(21.5, 0)

y-intercept(x=0): $f(0) = \sqrt{\frac{5}{2}} - 5$

$= \dfrac{\sqrt{10}}{2} - \dfrac{10}{2}$

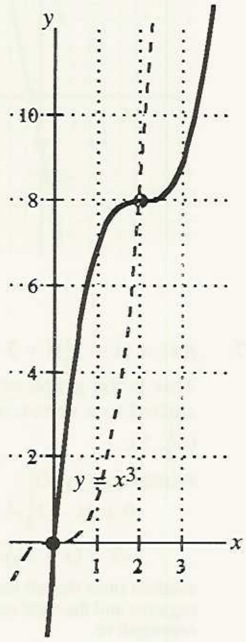
$= \dfrac{\sqrt{10} - 10}{2} \approx -3.4$



51. $f(x) = (x - 2)^3 + 8$
The graph of $y = x^3$ shifted to a new "origin" of (2, 8).

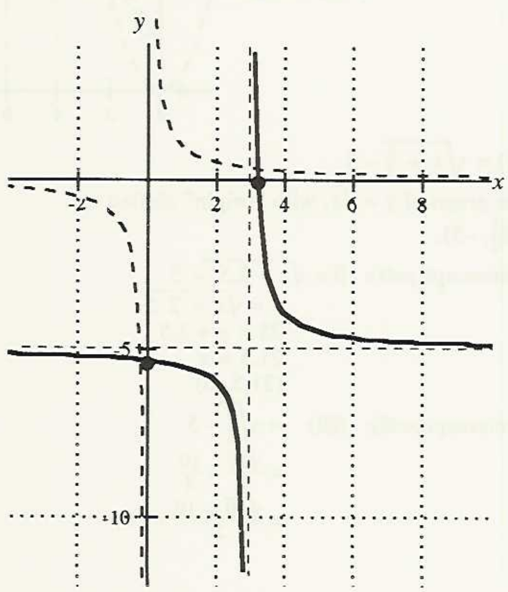
x-intercept(y=0):
$0 = \sqrt{x + 4} + 4$
$-4 = \sqrt{x + 4}$
No solution, so no x-intercept.

y-intercept(x=0):
$f(0) = 6$ (0, 6)

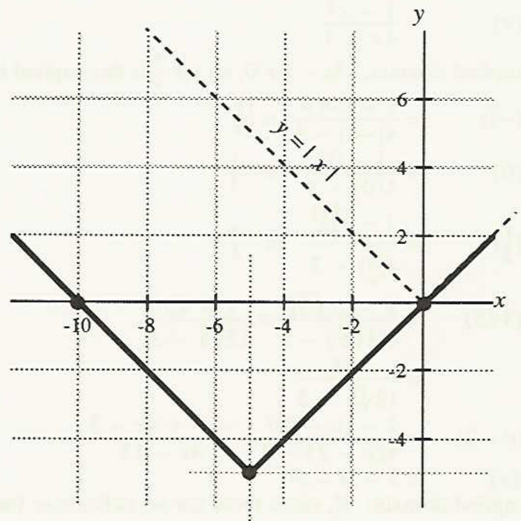


53. $f(x) = \dfrac{1}{x - 3} - 5$

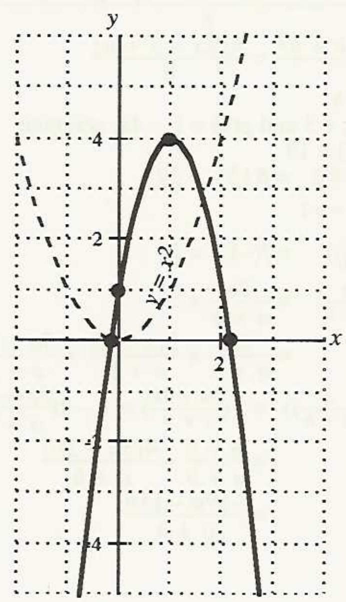
The graph of $y = \frac{1}{x}$ shifted to a new "origin" of (3, –5).
x-intercept(y=0):
$0 = \dfrac{1}{x - 3} - 5$
$5 = \dfrac{1}{x - 3}$
$5x - 15 = 1$
$x = \frac{16}{5}$ $(3\frac{1}{5}, 0)$
y-intercept(x=0): $f(0) = -\frac{1}{3} - 5 = -5\frac{1}{3}$ $(0, -5\frac{1}{3})$



55. $f(x) = | x + 5 | - 5$
The graph of $y = | x |$ shifted to a new "origin" of (–5, –5).
x-intercept(y=0): $0 = | x + 5 | - 5$
$5 = | x + 5 |$
$x + 5 = 5$ or $x + 5 = -5$
$x = 0$ or $x = -10$
(0, 0), (–10, 0)
y-intercept(x=0): $f(0) = 0$ (0, 0)



57. $f(x) = -3(x - 1)^2 + 4$
The graph of $y = x^2$, but flipped over, vertically scaled by 3, and origin shifted to (1, 4).
x-intercept(y=0): $0 = -3(x - 1)^2 + 4$
$-4 = -3(x - 1)^2$
$\frac{2}{3} = (x - 1)^2$
$x - 1 = \pm \frac{2\sqrt{3}}{3}$
$x = 1 \pm \frac{2\sqrt{3}}{3} \approx 2.2, -0.2$
(–0.2, 0), (2.2, 0)
y-intercept(x=0): $f(0) = 1$ (0, 1)



59. $f(x) = 2(x - 2)^3 + 1$

The graph of $y = x^3$, shifted to a new "origin" of (2, 1), and vertically scaled by 2 units.

x-intercept(y=0):

$0 = 2(x - 2)^3 + 1$

$-\frac{1}{2} = (x - 2)^3$

$x - 2 = \sqrt[3]{-\frac{1}{2}}$

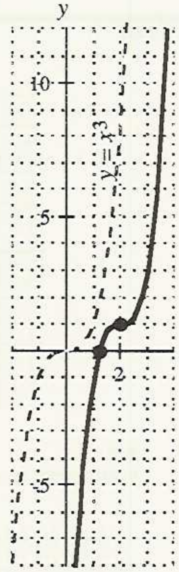
$x - 2 = \frac{\sqrt[3]{-1}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{-\sqrt[3]{4}}{\sqrt[3]{8}} = -\frac{\sqrt[3]{4}}{2}$

$x = 2 - \frac{\sqrt[3]{4}}{2} \qquad (2 - \frac{\sqrt[3]{4}}{2}, 0)$

y-intercept(x=0):

$f(0) = 2(-2)^3 + 1 = -15$

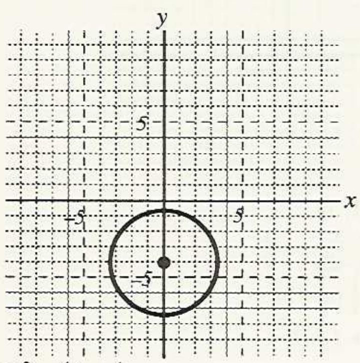
(0, –15)



61. $x^2 + (y + 4)^2 = 12$

$(x - 0)^2 + (y - (-4))^2 = 12$

Center: (0, –4); $r = \sqrt{12} = 2\sqrt{3} \approx 3.46$

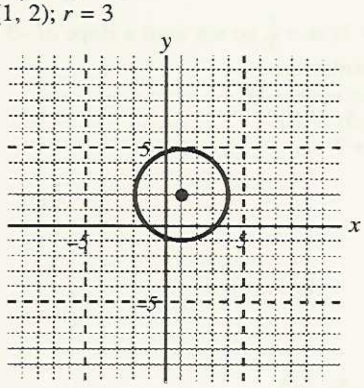


63. $x^2 - 2x + y^2 - 4y = 4$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 4 + 1 + 4$

$(x - 1)^2 + (y - 2)^2 = 9$

Center: (1, 2); $r = 3$



65. $(2x - 5)^2 + (2y + 3)^2 = 8$

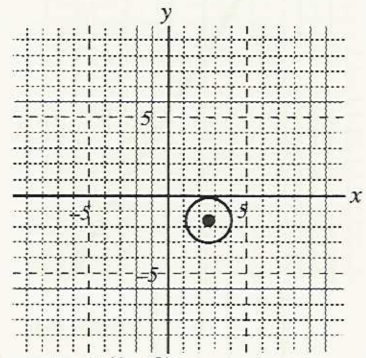
$[2(x - \frac{5}{2})]^2 + [2(y + \frac{3}{2})]^2 = 8$

$4(x - \frac{5}{2})^2 + 4(y + \frac{3}{2})^2 = 8$

$(x - \frac{5}{2})^2 + (y + \frac{3}{2})^2 = 2$

$(x - \frac{5}{2})^2 + (y - (-\frac{3}{2}))^2 = 2$

Center: $(2\frac{1}{2}, -1\frac{1}{2})$; radius = $\sqrt{2} \approx 1.4$



67. radius 3, center at (1, –3).

$(x - 1)^2 + (y - (-3))^2 = 3^2$

$(x - 1)^2 + (y + 3)^2 = 9$

69. Function, since it passes the vertical line test. Not one-to-one because it fails the horizontal line test.

71. Function, since it passes the vertical line test. One-to-one because it passes the horizontal line test.

73. $f(x) = \frac{-x}{x^2 + 1}$

$f(-x) = \frac{-(-x)}{(-x)^2 + 1} = \frac{x}{x^2 + 1}$

$-f(x) = -\frac{-x}{x^2 + 1} = \frac{x}{x^2 + 1}$

$f(-x) = -f(x)$, so the function is odd. It would have origin symmetry.

75. $h(x) = x\sqrt{x^2 - 3}$

$h(-x) = (-x)\sqrt{(-x)^2 - 3} = -x\sqrt{x^2 - 3} = -h(x)$.

Since $h(-x) = -h(x)$, this is an odd function, with symmetry about the origin.

77. $g(x) = \frac{3}{2 - x}$

$g(-x) = \frac{3}{2 - (-x)} = \frac{3}{2 + x}$

$-g(x) = \frac{-3}{2 - x}$ or $\frac{3}{-2 + x}$

The function is neither odd nor even, since $g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$.

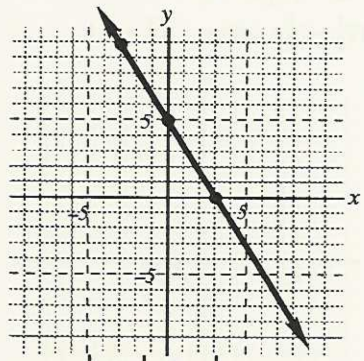
79. 40 and 55

81. 70; In generation 65 the frequencies are 8% and 77%, for a difference of 69%. In generation 70 the frequencies are 5% and 79%, for a difference of 74%.

Chapter 3 Test

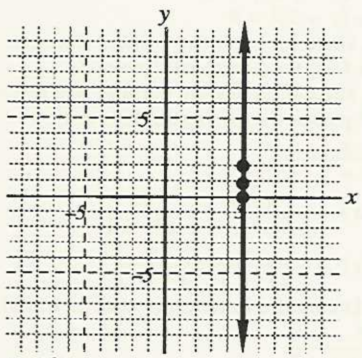
1. $3y + 5x = 15$
$y = -\frac{5}{3}x + 5$

x	y	
-3	10	
0	5	y-intercept
3	0	x-intercept



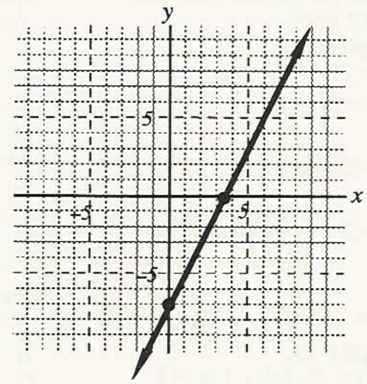
3. $x = 5$

x	y	
5	0	x-intercept
5	1	
5	2	

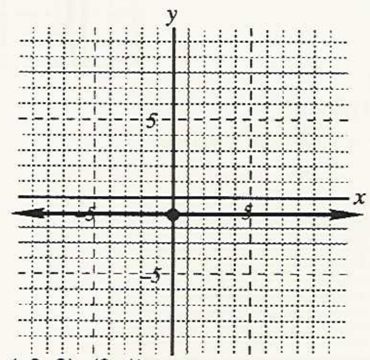


5. $(-2, 5\frac{1}{2}), (3, 2\frac{1}{2})$.
$$\left(\frac{-2+3}{2}, \frac{5\frac{1}{2}+2\frac{1}{2}}{2}\right) = (\tfrac{1}{2}, 4)$$

7. $2x - y = 7$
$-y = -2x + 7$
$y = 2x - 7$
$m = 2$
x-intercept (y=0): $2x = 7$
$x = 3\frac{1}{2}$
y-intercept (x=0): $y = -7$



9. $y = -1$
y-intercept is -1; no x-intercept.



11. $(-2, 3), (2, 1)$
$$m = \frac{1-3}{2-(-2)} = -\frac{1}{2}$$
$y - 1 = -\frac{1}{2}(x - 2)$
$y = -\frac{1}{2}x + 1 + 1$
$y = -\frac{1}{2}x + 2$

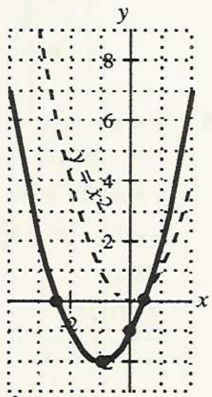
13. $y - \frac{1}{5}x = 1$
$y = \frac{1}{5}x + 1$; $m = \frac{1}{5}$, so we want a slope of -5 for a perpendicular line.
$y - (-3) = -5(x - 2)$
$y + 3 = -5x + 10$
$y = -5x + 7$

15. Let (x, y) be a point equidistant from the points (2, 3) and (4, 6).
Distance from (x, y) to (2, 3) = Distance from (x, y) to (4, 6)
$\sqrt{(x-2)^2 + (y-3)^2} = \sqrt{(x-4)^2 + (y-6)^2}$
$(x-2)^2 + (y-3)^2 = (x-4)^2 + (y-6)^2$
$4x + 6y - 39 = 0$

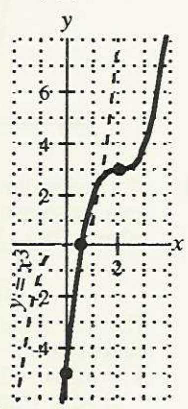
17. $f(x) = \dfrac{x}{x-3}$
Domain: $x - 3 \neq 0$, so $x \neq 3$ is the domain.
$f(-2) = \dfrac{-2}{-2-3} = \dfrac{2}{5}$
$f(0) = \dfrac{0}{0-3} = 0$
$f(3)$; 3 is not in the domain of f.
$f(c-3) = \dfrac{c-3}{(c-3)-3} = \dfrac{c-3}{c-6}$

19. $v(x) = 3 - 2x - x^2$
No restrictions on x, so the domain is all real numbers R.
$v(-2) = 3 - 2(-2) - (-2)^2 = 3$
$v(0) = 3 - 2(0) - (0)^2 = 3$
$v(3) = 3 - 2(3) - (3)^2 = -12$
$v(c-3) = 3 - 2(c-3) - (c-3)^2 = 4c - c^2$

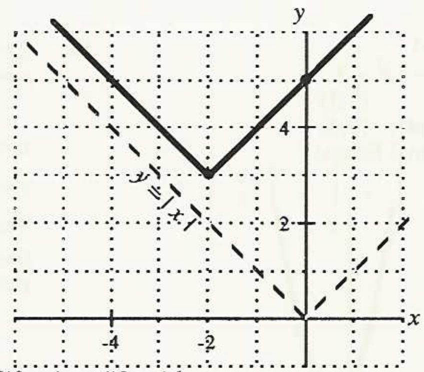
21. $f(x) = (x+1)^2 - 2$
This is the graph of $y = x^2$ shifted to a vertex of $(-1, -2)$.
Vertex at $(-1, -2)$.
x-intercept (y=0): $0 = (x+1)^2 - 2$
$2 = (x+1)^2$
$\pm\sqrt{2} = x + 1$
$x = -1 \pm \sqrt{2} \approx -2.4, -0.4$
$(-2.4, 0), (-0.4, 0)$
y-intercept (x=0): $f(0) = 1^2 - 2 = -1$



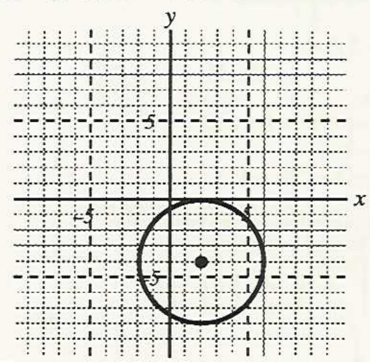
23. $f(x) = (x-2)^3 + 3$
The graph of $y = x^3$ with the origin shifted to $(2, 3)$.
x-intercept (y=0): $0 = (x-2)^3 + 3$
$-3 = (x-2)^3$
$\sqrt[3]{-3} = x - 2$
$2 + \sqrt[3]{-3} = x$
$0.6 \approx x$
$(0.6, 0)$
y-intercept (x=0): $f(0) = -5$



25. $f(x) = |x+2| + 3$
This is the graph of $y = |x|$, with origin shifted to $(-2, 3)$.
x-intercept (y=0): $0 = |x+2| + 3$
$-3 = |x+2|$
No solution, since $|z| \geq 0$.
y-intercept (x=0): $f(0) = 5$.



27. $(x-2)^2 + (y+4)^2 = 16$
$(x-2)^2 + (y-(-4))^2 = 16$
Center: $(2, -4)$; radius $= \sqrt{16} = 4$



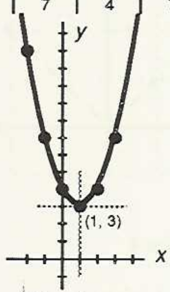
29. radius $4\sqrt{5}$, center at $(-3\frac{1}{2}, 2)$.
$(x - (-3\frac{1}{2}))^2 + (y-2)^2 = (4\sqrt{5})^2$
$(x + \frac{7}{2})^2 + (y-2)^2 = 80$
31. (a) -2 ; (b) 4 ; (c) -3 ; (d) -3
33. $-8, -1, 4, 9$
35. -9 to -8, -4 to 1.5, 6 to 10
37. -3.5 to 4
39. $h(x) = \dfrac{-5x}{x - x^2}$
$h(-x) = \dfrac{-5(-x)}{(-x) - (-x)^2} = \dfrac{5x}{-x - x^2} = \dfrac{5x}{-(x + x^2)} = \dfrac{-5x}{x + x^2}$
$-h(x) = \dfrac{5x}{x - x^2}$
Since $h(-x) \neq h(x)$, and $h(-x) \neq -h(x)$, this function is neither even nor odd, and does not have y-axis or origin symmetry.
41. (a) Use (altitude, feet) points (5060, 94.9) and (5510, 94.4) to obtain a line $y = -\dfrac{x}{900} + 100.52$. Using x= 5280 we obtain $y \approx 94.7°$ Celcius.
(b) Use (altitude, feet) points (10320, 89.8) and (15430, 84.9) to obtain a line $y = -\dfrac{7x}{7300} + 99.696$. Using x= 12,000 we obtain $y \approx 88.2°$ Celcius .

Chapter 4

Section 4–1

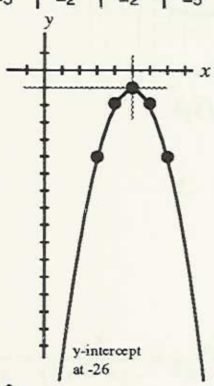
1. $y = (x-1)^2 + 3$
Vertex (1,3)
Intercepts: (0,4)
Additional Points:

x	-2	-1	2	3
y	12	7	4	7

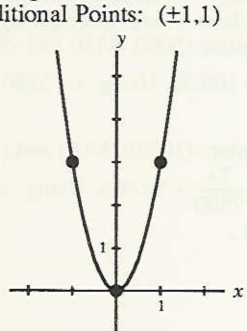


5. $y = -(x-5)^2 - 1$
Vertex (5,–1)
Intercepts:
$x=0$: $y = -(-5)^2 - 1 = -26$
(0,–26)
$y=0$: $0 = -(x-5)^2 - 1$
$1 = -(x-5)^2$; no real solution since the left side is positive and the right side is negative. Thus, no x–intercepts.
Additional Points:

x	3	4	6	7
y	-5	-2	-2	-5



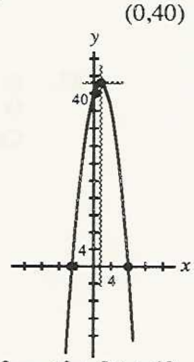
9. $y = 3x^2$
Vertex (0,0)
Intercepts: (0,0)
Additional Points: ($\pm$1,1)



Additional Points:

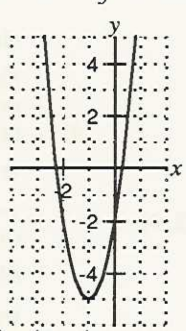
x	-1	1	2
y	3	1	6

13. $y = -x^2 + 3x + 40$
$y = -(x^2 - 3x) + 40$
$\frac{1}{2}\cdot(-3) = -\frac{3}{2}$; $(-\frac{3}{2})^2 = \frac{9}{4}$
$y = -(x^2 - 3x + \frac{9}{4}) + 40 + 1(\frac{9}{4})$
$y = -(x - \frac{3}{2})^2 + 42\frac{1}{4}$
Vertex $(1\frac{1}{2}, 42\frac{1}{4})$
Intercepts:
$x=0$: $y = -0^2 + 0 + 40 = 40$
(0,40)

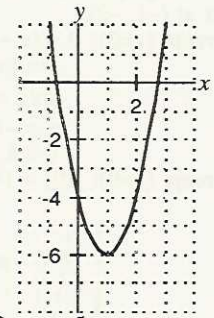


$y=0$: $0 = -x^2 + 3x + 40$
$0 = x^2 - 3x - 40$
$0 = (x-8)(x+5)$
$x = -5$ or 8 (–5,0), (8,0)

17. $y = 3x^2 + 6x - 2$
$y = 3(x^2 + 2x) - 2$
$y = 3(x^2 + 2x + 1) - 2 - 3(1)$
$y = 3(x+1)^2 - 5$
Vertex (–1,–5)
Intercepts:
$x=0$: $y = 3\cdot0^2 + 6\cdot0 - 2 = -2$
(0,–2)
$y=0$: $0 = 3(x+1)^2 - 5$
$\frac{5}{3} = (x+1)^2$
$x + 1 = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}$
$x = -1 \pm \frac{\sqrt{15}}{3} \approx -2.3, 0.3$

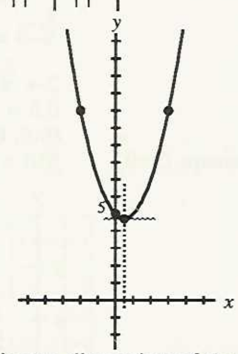


21. $y = 2x^2 - 4x - 4$
$y = 2(x^2 - 2x) - 4$
$y = 2(x^2 - 2x + 1) - 4 - 2$
$y = 2(x-1)^2 - 6$
Vertex (1,–6)
Intercepts:
$x=0$: $y = 0 - 0 - 4 = -4$ (0,–4)
$y=0$: $0 = 2(x-1)^2 - 6$
$(x-1)^2 = 3$
$x - 1 = \pm\sqrt{3}$
$x = 1 \pm \sqrt{3}$
(0.7,0), (2.7,0)

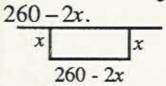


25. $y = x^2 - x + 5$
$\frac{1}{2}\cdot(-1) = -\frac{1}{2}$; $(-\frac{1}{2})^2 = \frac{1}{4}$
$y = x^2 - x + \frac{1}{4} + 5 - \frac{1}{4}$
$y = (x - \frac{1}{2})^2 + 4\frac{3}{4}$
Vertex $(\frac{1}{2}, 4\frac{3}{4})$
Intercepts:
$x=0$: $y = 0 - 0 + 5 = 5$ (5,0)
$y=0$: $0 = x^2 - x + 5$
$x = \frac{1 \pm \sqrt{-19}}{2}$
Not real — no x–intercepts.
Additional Points:

x	-2	3
y	11	11



29. If x is one dimension of the rectangular area, as shown in the figure, then the other is $260 - 2x$.



The area is the product of these two dimensions:
$A = x(260 - 2x)$
$= -2x^2 + 260x$
$= -2(x^2 - 130x)$
$= -2(x^2 - 130x + 4225) + 2(4225)$
$A = -2(x - 65)^2 + 8450$
Vertex: $(65,8450) = (x, A)$.
Since this is a parabola which opens downwards, the vertex is the highest point, corresponding to the maximum area. This is where $x = 65$ ft, $260 - 2x = 130$ ft, and $A = 8{,}450$ ft^2.

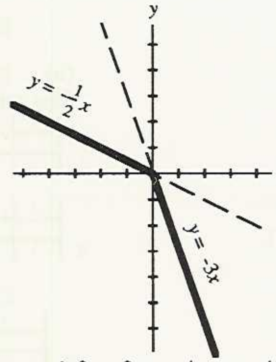
33. Since $v_0 = 64$,
$s = 64t - 16t^2$ Using $v_o = 64$
$= -16(t^2 - 4t)$
$= -16(t^2 - 4t + 4) + 16(4)$
$= -16(t-2)^2 + 64$
Vertex: $(2,64) = (t, s)$.
Thus the maximum height of $s = 64$ feet is reached after $t = 2$ seconds.

If $s = 0$, $0 = 64t - 16t^2 = 16t(4 - t)$, so $t = 0$ or 4. 0 corresponds to when the object is thrown, and it returns to earth after 4 seconds.

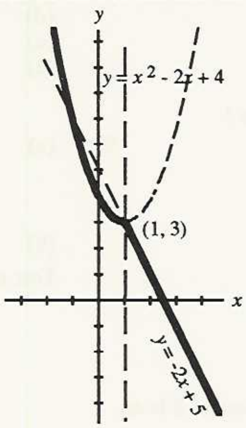
37. $P = -u^2 + 100u - 1000$
$= -(u^2 - 100u + 2500) - 1000 + 2500$
$= -(u - 50)^2 + 1500$
Vertex: $(50,1500) = (u, P)$.
Thus a production of 50 units will produce the maximum profit of \$1500.

41. The dimensions of a rectangle with perimeter P are x and $\frac{1}{2}(P - 2x) = \frac{P}{2} - x$. Thus the area is $A = x(\frac{P}{2} - x) = \frac{P}{2}x - x^2$, a parabola which opens downwards.
$A = -x^2 + \frac{P}{2}x$
$= -(x^2 - \frac{P}{2}x)$
$= -(x^2 - \frac{P}{2}x + \frac{P^2}{16}) + \frac{P^2}{16}$
$= -(x - \frac{P}{4})^2 + \frac{P^2}{16}$
Vertex: $(\frac{P}{4}, \frac{P^2}{16}) = (x, A)$.
The maximum is at the vertex where $x = \frac{P}{4}$, and $A = \frac{P^2}{16}$.
The circumference (perimeter) of a circle is $C = 2\pi r$, so if the perimeter is P, $P = 2\pi r$, or $r = \frac{P}{2\pi}$. The area $A = \pi r^2 = \pi(\frac{P}{2\pi})^2 = \pi(\frac{P^2}{4\pi^2}) = \frac{P^2}{4\pi}$. Since $4\pi < 16$, $\frac{P^2}{4\pi} > \frac{P^2}{16}$, and the circle will have a larger area.

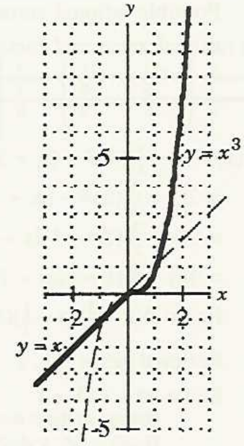
45. $g(x) = \begin{cases} -\frac{1}{2}x, & x < 0 \\ -3x, & x \geq 0 \end{cases}$
•Graph $y = -\frac{1}{2}x$: Intercepts at the origin; also plot an additional point, say $(-2,1)$.
•Graph $y = -3x$: Intercepts at the origin; also plot an additiona point, say $(1,-3)$.
•Darken in the first line for $x < 0$; darken in the second line for $x \geq 0$.



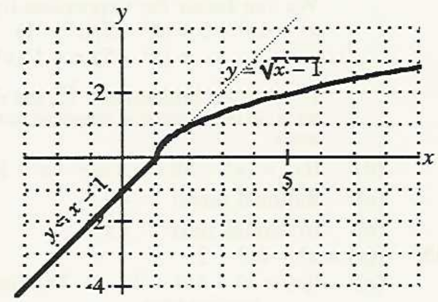
49. $h(x) = \begin{cases} x^2 - 2x + 4, & x < 1 \\ -2x + 5, & x \geq 1 \end{cases}$
•Graph $y = x^2 - 2x + 4$:
$y = x^2 - 2x + 1 + 4 - 1$
$y = (x - 1)^2 + 3$
Parabola with Vertex at (1,3); y–intercept at $y = 3$.
•Graph $y = -2x + 5$:
x–intercept: $x = 2\frac{1}{2}$
y–intercept: $y = 5$
•Darken in the first line for $x < 1$; darken in the second line for $x \geq 1$.



53. $g(x) = \begin{cases} x, & x < 0 \\ x^3, & x \geq 0 \end{cases}$



57. $g(x) = \begin{cases} x - 1, & x < 1 \\ \sqrt{x - 1}, & x \geq 1 \end{cases}$



Exercise 4–2

1. $f(x) = 7$
Replace $f(x)$ by 0: $0 = 7$; no solutions, so no zeros.

5. $h(x) = x + 11$
Replace $h(x)$ by 0: $0 = x + 11$; $x = -11$

9. $x^4 - 3x^2 + 6$: In $\frac{p}{q}$ p divides 6 and q divides 1, so we have $\pm\frac{6}{1}, \pm\frac{3}{1}, \pm\frac{2}{1}, \pm\frac{1}{1}$ or $\pm6, \pm3, \pm2, \pm1$.

13. $6x^2 - 5 + 2x^3$; rewrite as $2x^3 + 6x^2 - 5$: In $\frac{p}{q}$ p divides 5 and q divides 2, so we have $\pm\frac{5}{2}, \pm\frac{5}{1}, \pm\frac{1}{2}, \pm\frac{1}{1}$, or $\pm\frac{5}{2}, \pm\frac{1}{2}, \pm5, \pm1$.

17. $5x^4 - 2x^3 + 3x - 10$: In $\frac{p}{q}$ p divides 10 and q divides 5, so we have $\pm\frac{10}{5}, \pm\frac{10}{1}, \pm\frac{5}{5}, \pm\frac{5}{1}, \pm\frac{2}{5}, \pm\frac{2}{1}, \pm\frac{1}{5}, \pm\frac{1}{1}$ or $\pm10, \pm5, \pm2, \pm1, \pm\frac{1}{5}, \pm\frac{2}{5}$.

21. $2 - 3x^2 + 4x^3$; rewrite as $4x^3 - 3x^2 + 2$: In $\frac{p}{q}$ p divides 2 and q divides 4, so we have $\pm\frac{2}{4}, \pm\frac{2}{2}, \pm\frac{2}{1}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{1}{1}$ or $\pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm1$.

25. $8x^3 - 8x + 16$; rewrite as $8(x^3 - x + 2)$ and focus on $x^3 - x + 2$. In $\frac{p}{q}$ p divides 2 and q divides 1, so we have $\pm\frac{2}{1}, \pm\frac{1}{1}$ or $\pm2, \pm1$.

29. (a)

	1	−2	3	1
		1	−1	2
1	1	−1	2	3

$x^2 - x + 2 + \frac{3}{x - 1}$

(b) $f(1) = 3$.

33. (a)

	$\frac{1}{2}$	−3	$\frac{3}{4}$	−3
		3	0	$\frac{9}{2}$
6	$\frac{1}{2}$	0	$\frac{3}{4}$	$\frac{3}{2}$

$\frac{1}{2}x^2 + \frac{3}{4} + \frac{\frac{3}{2}}{x - 6}$

(b) $f(6) = \frac{3}{2}$

37. (a) $f(x) = 4x^3 - 12x^2 + 11x - 3$ There are 3 sign changes so there are 1 or 3 positive zeros.
$f(-x) = -4x^3 - 12x^2 - 11x - 3$ There are no sign changes so there are no negative zeros.

(b) Possible rational zeros: $\frac{3}{4}, \frac{3}{2}, \frac{1}{4}, \frac{1}{2}, 3, 1$.

Test rational zeros and factor.

	4	−12	11	−3
		6	−9	3
$\frac{3}{2}$	4	−6	2	0

$(x - \frac{3}{2})$ is a factor of f(x).

$f(x) = (x - \frac{3}{2})(4x^2 - 6x + 2)$

$= 2(x - \frac{3}{2})(2x^2 - 3x + 1)$ Factor 2 from $4x^2 - 6x + 2$.

$= 2(x - \frac{3}{2})(2x - 1)(x - 1)$ Factor $2x^2 - 3x + 1$.

$= 4(x - \frac{3}{2})(x - \frac{1}{2})(x - 1)$ $(2x - 1) = 2(x - \frac{1}{2})$

(d) $f(x) = 4(x - \frac{3}{2})(x - \frac{1}{2})(x - 1)$.

(c) Rational zeros $\frac{3}{2}, \frac{1}{2}, 1$ From the factors of part (d)

41. (a) $f(x) = x^6 - 4x^3 - 5$ There is one sign change so there is one positive zero.

$f(-x) = x^6 + 4x^3 - 5$ There is one sign change so there is one negative zero.

(b) Possible rational zeros: $\pm 1, \pm 5$

We can factor the expression for f(x).

$x^6 - 4x^3 - 5 = (x^3 - 5)(x^3 + 1)$

$= (x^3 - 5)(x + 1)(x^2 - x + 1)$

$x^3 - 5$ has the irrational zero $\sqrt[3]{5}$, and $x^2 - x + 1$ has only complex zeros, so this expression cannot be factored further using rational zeros.

(d) $f(x) = (x^3 - 5)(x + 1)(x^2 - x + 1)$.

(c) Rational zeros: -1

(e) Irrational zero: $\sqrt[3]{5}$

45. $f(x) = x^5 - 4x^2 + 2$

(a) $f(x) = x^5 - 4x^2 + 2$ Two sign changes; 0 or 2 positive zeros.

$f(-x) = -x^5 - 4x^2 + 2$ One sign change; one negative zero.

(b) Possible rational zeros: $\pm 1, \pm 2$.

Test rational zeros and factor.

	1	0	0	−4	0	2
		1	1	1	−3	−3
1	1	1	1	−3	−3	−1

	1	0	0	−4	0	2
		2	4	8	8	16
2	1	2	4	4	8	18

Third row all positive so 2 is an upper bound for positive zeros.

We have checked all the possible positive rational zeros; 2 is an upper bound for positive zeros (whereas 1 is not).

We now check for negative rational zeros.

	1	0	0	−4	0	2
		−1	1	−1	5	−5
−1	1	−1	1	−5	5	−3

Alternating signs in last row so −1 is a lower bound for negative zeros.

(e) Thus, f has one negative irrational zero between −1 and 0. It has 0 or 2 positive irrational zeros between 0 and 2.

49. (a) $f(x) = 3x^4 + 2x^3 - 4x^2 - 2x + 1$ Two sign changes; 0 or 2 positive zeros.

$f(-x) = 3x^4 - 2x^3 - 4x^2 + 2x + 1$ Two sign changes; 0 or 2 negative zeros.

(b) Possible rational zeros: $\pm\frac{1}{3}, \pm 1$

Test rational zeros and factor.

	3	2	−4	−2	1
		3	5	1	−1
1	3	5	1	−1	0

$(x - 1)$ is a factor of f(x).

$f(x) = (x - 1)(3x^3 + 5x^2 + x - 1)$

	3	5	1	−1
		−3	−2	1
−1	3	2	−1	0

$(x + 1)$ is a factor of f(x).

$f(x) = (x - 1)(x + 1)(3x^2 + 2x - 1)$

$f(x) = (x - 1)(x + 1)(3x - 1)(x + 1)$

$f(x) = 3(x - 1)(x + 1)^2(x - \frac{1}{3})$

(d) $f(x) = 3(x - 1)(x + 1)^2(x - \frac{1}{3})$.

(c) Rational zeros: 1, −1 (mult 2), $\frac{1}{3}$

53. $f(x) = x^5 - 4x^3 + x^2 + 4x + 4$

(a) $f(x) = x^5 - 4x^3 + x^2 + 4x + 4$ Two sign changes; 0 or 2 positive zeros.

$f(-x) = -x^5 + 4x^3 + x^2 - 4x + 4$ Three sign changes; 1 or 3 negative zeros.

(b) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

Test rational zeros.

	1	0	−4	1	4	4
		1	1	−3	−2	2
1	1	1	−3	−2	2	6

	1	0	−4	1	4	4
		2	4	0	2	12
2	1	2	0	1	6	15

Last row is nonnegative, so 2 is an upper bound for positive zeros.

Checking −1 shows it is not a zero or a lower bound.

	1	0	−4	1	4	4
		−2	4	0	−2	−4
−2	1	−2	0	1	2	0

$f(x) = (x + 2)(x^4 - 2x^3 + x + 2)$

Focus on the expression $g(x) = x^4 - 2x^3 + x + 2$.

$g(x) = x^4 - 2x^3 + x + 2$ Two sign changes; 0 or 2 positive zeros.

$g(-x) = x^4 + 2x^3 - x + 2$ Two sign changes; 0 or 2 negative zeros.

Possible zeros of $x^4 - 2x^3 + x + 2$ are $\pm 1, \pm 2$. However if it had a positive rational zero this value would be a zero of f(x) also. Thus there are no positive rational zeros of $x^4 - 2x^3 + x + 2$.

	1	−2	0	1	2
		−1	3	−3	2
−1	1	−3	3	−2	4

The alternating signs in the last row indicate −1 is a lower bound for negative zeros.

(d) $f(x) = (x + 2)(x^4 - 2x^3 + x + 2)$

(c) Rational zeros: −2

(e) There are 0 or 2 positive irrational zeros between the values 0 and 2. There are 0 or 2 negative irrational zeros between the values 0 and −1.

57. (a) $f(x) = 4x^5 + 16x^4 + 37x^3 + 43x^2 + 22x + 4$
No sign changes; no positive zeros.

$f(-x) = -4x^5 + 16x^4 - 37x^3 + 43x^2 - 22x + 4$
Five sign changes; 1, 3 or 5 negative zeros.

(b) Possible rational zeros: $-\frac{1}{4}, -\frac{1}{2}, -1, -2, -4$

Test rational zeros and factor.

	4	16	37	43	22	4
		−4	−12	−25	−18	−4
−1	4	12	25	18	4	0

$f(x) = (x + 1)(4x^4 + 12x^3 + 25x^2 + 18x + 4)$

	4	12	25	18	4
		−2	−5	−10	−4
$-\frac{1}{2}$	4	10	20	8	0

$f(x) = (x + 1)(x + \frac{1}{2})(4x^3 + 10x^2 + 20x + 8)$

$f(x) = (2)(x + 1)(x + \frac{1}{2})(2x^3 + 5x^2 + 10x + 4)$

	2	5	10	4
		−1	−2	−4
$-\frac{1}{2}$	2	4	8	0

$f(x) = 2(x + 1)(x + \frac{1}{2})^2(2x^2 + 4x + 8)$

$f(x) = 2(2)(x + 1)(x + \frac{1}{2})^2(x^2 + 2x + 4)$

$x^2 + 2x + 4$ is prime on R.

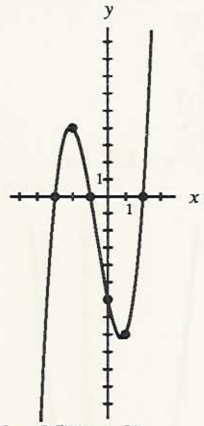
(d) $f(x) = 4(x + 1)(x + \frac{1}{2})^2(x^2 + 2x + 4)$.

(c) Rational zeros: $-\frac{1}{2}$ (mult 2), −1.

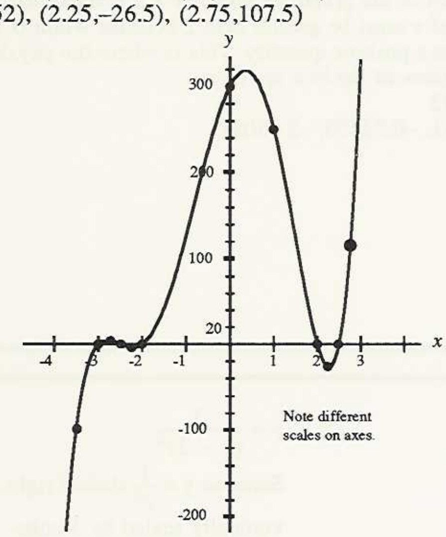
61. If $\frac{p}{q}$ is a rational zero then b, d, or e divides p and a or c divides q. Thus, possible numerators are 1, b, d, e, bd, be, de, bde, and possible denominators are a, c, and ac. Thus the possible rational zeros are $\pm\frac{1}{a}, \pm\frac{1}{c}, \pm\frac{1}{ac}, \pm\frac{b}{a}, \pm\frac{d}{a}, \pm\frac{e}{a}, \pm\frac{bd}{a}, \pm\frac{be}{a}$, $\pm\frac{de}{a}, \pm\frac{bde}{a}, \pm\frac{b}{c}, \pm\frac{d}{c}, \pm\frac{e}{c}, \pm\frac{bd}{c}, \pm\frac{be}{c}, \pm\frac{de}{c}, \pm\frac{bde}{c}$, $\pm\frac{b}{ac}, \pm\frac{d}{ac}, \pm\frac{e}{ac}, \pm\frac{bd}{ac}, \pm\frac{be}{ac}, \pm\frac{de}{ac}, \pm\frac{bde}{ac}, \pm b, \pm d, \pm e, \pm bd, \pm be, \pm de$, $\pm bde$.

Exercise 4-3

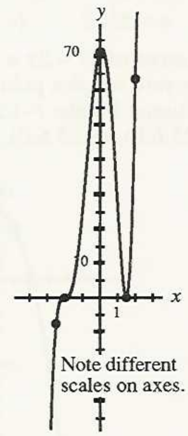
1. $f(x) = (x - 2)(x + 1)(x + 3)$
   $y = (x - 2)(x + 1)(x + 3)$
   Intercepts:
   $x=0$: $y = (-2)(1)(3) = -6$ $(0,-6)$
   $y=0$: $0 = (x - 2)(x + 1)(x + 3)$
   $x - 2 = 0$ or $x + 1 = 0$ or $x + 3 = 0$
   $x = -3, -1, 2$
   $(-3,0), (-1,0), (2,0)$
   Additional Points: $(-2,4), (1,-8)$



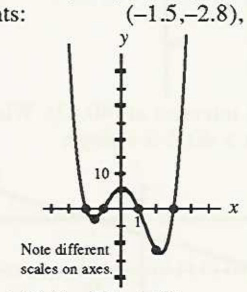
5. $g(x) = (x^2 - 4)(4x^2 - 25)(x + 3)$
   $y = (x-2)(x+2)(2x-5)(2x+5)(x+3)$
   Intercepts:
   $x=0$: $y = (-4)(-25)(3) = 300$ $(300,0)$
   $y=0$: $0 = (x-2)(x+2)(2x-5)(2x+5)$
   $(x+3)$; $x = \pm 2, \pm\frac{5}{2}, -3$
   $(-3,0), (-2.5,0), (-2,0), (2,0), (2.5,0)$
   Additional Points:
   $(-3.5,-99), (-2.75, 4.7), (-2.25,-3.8)$
   $(1,252), (2.25,-26.5), (2.75,107.5)$



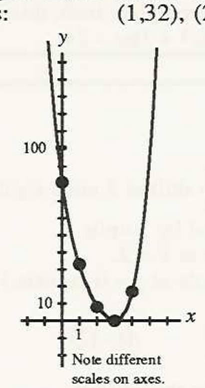
9. $h(x) = (x + 2)^3(2x - 3)^2$
   $y = (x + 2)^3(2x - 3)^2$
   Intercepts:
   $x=0$: $y = 2^3(-3)^2 = 72$ $(0,72)$
   $y=0$: $0 = (x + 2)^3(2x - 3)^2$
   $x = -2$ (multiplicity 3), $(-2,0)$
   $\frac{3}{2}$ (multiplicity 2) $(1\frac{1}{2},0)$
   Additional Points:
   $(-2.5,-8), (2,64)$



13. $f(x) = x^4 - x^3 - 7x^2 + x + 6$
   Using synthetic division with potential values for rational zeros we find that
   $y = (x - 3)(x - 1)(x + 1)(x + 2)$.
   Intercepts:
   $x=0$: $y = 0^4 - 0^3 - 7(0)^2 + 0 + 6 = 6$ $(0,6)$
   $y=0$: $0 = (x - 3)(x - 1)(x + 1)(x + 2)$
   $x = -2, -1, 1, 3$
   $(-2,0), (-1,0), (1,0), (3,0)$
   Additional Points: $(-1.5,-2.8), (2,-12)$



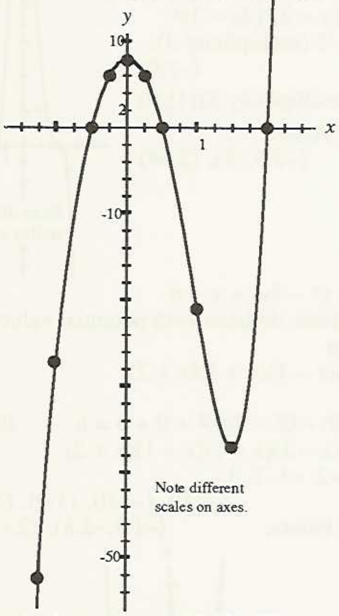
17. $f(x) = x^4 - 8x^3 + 30x^2 - 72x + 81$
   Potential rational zeros and synthetic division produce:
   $y = (x - 3)^2(x^2 - 2x + 9)$. $x^2 - 2x + 9$ prime on R.
   Intercepts:
   $x=0$: $y = 0 - 0 + 0 - 0 + 81 = 81$ $(0,81)$
   $y=0$: $0 = (x - 3)^2(x^2 - 2x + 9)$
   $x = 3$ (multiplicity 2) $(3,0)$
   The zeros of $x^2 - 2x + 9$ are $1 \pm 2\sqrt{2}i$; their real component is 1. Therefore we plot values around this value.
   Additional values: $(1,32), (2,9), (4,17)$



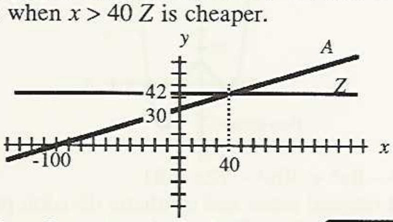
21. $h(x) = 4x^5 - x^3 - 32x^2 + 8$
   The expression can be factored by grouping.
   $y = x^3(4x^2 - 1) - 8(4x^2 - 1)$
   $= (4x^2 - 1)(x^3 - 8)$
   $= (2x-1)(2x+1)(x-2)(x^2+2x+4)$

Intercepts:
x=0: $y = 8$ (0,8)
y=0: $0 = (2x - 1)(2x + 1)(x - 2)(x^2 + 2x + 4)$
$x = \pm\frac{1}{2}, 2$ $(-\frac{1}{2},0)$, $(\frac{1}{2},0)$, (2,0)

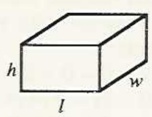
The zeros of $x^2 + 2x + 4$ are $-1 \pm \sqrt{3}i$, with real part –1. We make sure we plot points around –1.
Additional Points: (–1.25,–52.2), (–1,–27), (–0.25,6.0), (0.25,6.0), (1,–21), (1.5,–37)



25. (b) The graphs intersect at (40,42). When $x < 40$ A is cheaper; when $x > 40$ Z is cheaper.



29. The total surface area A of a box with length l, width w and height h is $A = 2hl + 2lw + 2wh$.

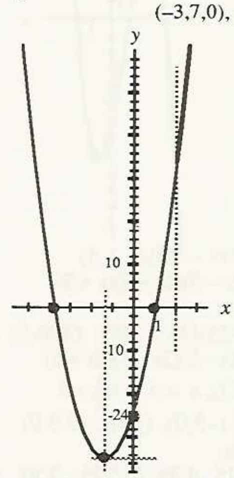


Using $l = x + 6$, $w = x - 2$ and $h = x$ we obtain
$A(x) = 2x(x + 6) + 2(x + 6)(x - 2) + 2x(x - 2)$
$= 6x^2 + 16x - 24$ in^2.
With a cost of 1¢ per square inch, this is also the cost function.
We graph $y = 6x^2 + 16x - 24$.

$y = 6(x^2 + \frac{8}{3}x) - 24$
$y = 6(x^2 + \frac{8}{3}x + \frac{16}{9}) - 24 - 6(\frac{16}{9})$
$y = 6(x + \frac{4}{3})^2 - \frac{104}{3}$.

Parabola; vertex: $(-1\frac{1}{3},-34\frac{2}{3})$

Intercepts:
x=0: $y = 6x^2 + 16x - 24$
$y = 0 + 0 - 24 - 24$ (0,–24)
y=0: $y = 6(x + \frac{4}{3})^2 - \frac{104}{3}$
$0 = 6(x + \frac{4}{3})^2 - \frac{104}{3}$
$6(x + \frac{4}{3})^2 = \frac{104}{3}$
$(x + \frac{4}{3})^2 = \frac{52}{9}$
$x + \frac{4}{3} = \pm\sqrt{\frac{52}{9}}$
$x = -\frac{4}{3} \pm \frac{2\sqrt{13}}{3} \approx -3{,}7, 1.1$
(–3,7,0), (1.1,0)



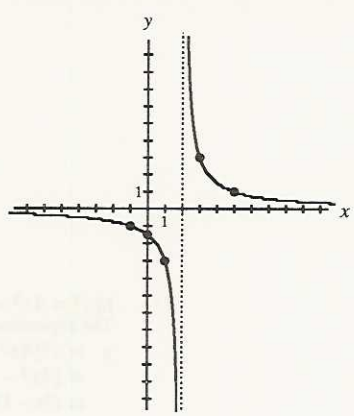
The part of the graph for which $x > 2$ is darkened in. The value of x must be greater than 2 because width is $x - 2$, and must be a positive quantity. This is where the physical restrictions of the box are met.

33. 0.46682
37. 0.69091, –0.72720, –2.65081

Exercise 4–4

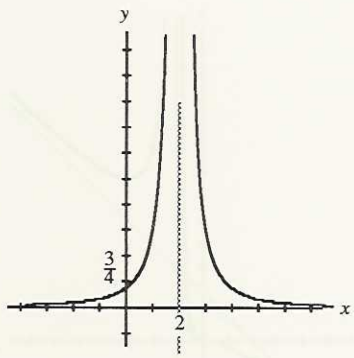
1. $y = \dfrac{3}{x - 2}$

The graph of $y = \frac{1}{x}$ shifted 2 units right and vertically scaled by 3 units.
Vertical asymptote at $x = 2$.
Horizontal asymptote at $y = 0$ (x–axis.)
Intercepts:
x=0: $y = \frac{3}{-2} = -1\frac{1}{2}$ $(0,-1\frac{1}{2})$
y=0: $0 = \dfrac{3}{x - 2}$ has no solution.
Additional Points: (–1,–1), (1,–3), (3,3), (5,1)



5. $y = \dfrac{3}{(x - 2)^2}$

Same as $y = \frac{1}{x^2}$ shifted right 2 units and vertically scaled by 3 units.
Vertical asymptote at $x = 2$.
Horizontal asymptote at $y = 0$ (x–axis.)
Intercepts:
x=0: $y = \dfrac{3}{(-2)^2} = \dfrac{3}{4}$ $(0,\frac{3}{4})$
y=0: $0 = \dfrac{3}{(x - 2)^2}$ has no solution.
Additional Points: $(-1,\frac{1}{3})$, (1,3), (3,3), $(4,\frac{3}{4})$



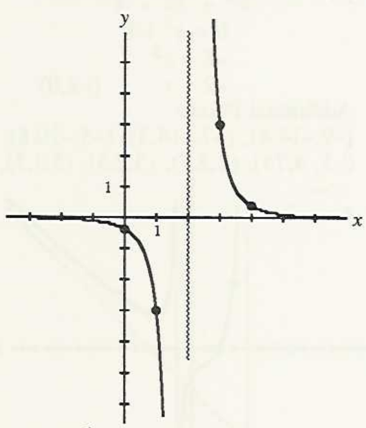
9. $y = \dfrac{3}{(x-2)^3}$

Same as $y = \dfrac{1}{x^3}$ translated 2 units right, vertically scaled 3 units. Vertical asymptote at $x = 2$.
Horizontal asymptote at $y = 0$ (x–axis.)
Intercepts:

x=0: $y = \dfrac{3}{(-2)^3} = -\frac{3}{8}$ $\quad (0,-\frac{3}{8})$

y=0: $0 = \dfrac{3}{(x-2)^3}$ has no solutions.

Additional Points: (1,–3), (3,3), $(4,\frac{3}{8})$



13. $y = \dfrac{1}{x-1} + 2$

Same as $y = \dfrac{1}{x}$ translated 1 unit right and 2 units up.
Vertical asymptote at $x = 1$.
Horizontal asymptote at $y = 2$.
"Origin": (1,2)
Intercepts:

x=0: $y = \dfrac{1}{-1} + 2 = 1$ $\quad (0,1)$

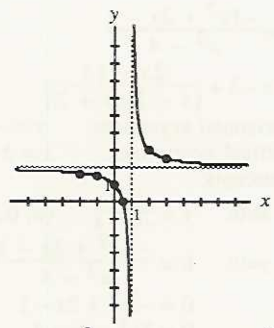
y=0: $0 = \dfrac{1}{x-1} + 2$

$-2 = \dfrac{1}{x-1}$

$-x + 2 = 1$

$1 = x$ $\quad (1,0)$

Additional Points:
$(-2,1\frac{2}{3})$, $(-1,1\frac{1}{2})$, (2,3), $(3,2\frac{1}{2})$



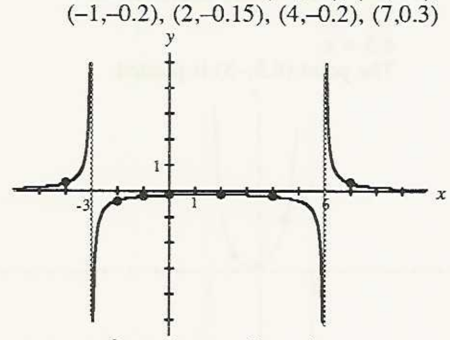
17. $y = \dfrac{3}{x^2 - 3x - 18} = \dfrac{3}{(x-6)(x+3)}$

Vertical asymptotes at $x = -3, 6$.
Horizontal asymptote at $y = 0$ (x–axis.)
Intercepts:

x=0: $y = \dfrac{3}{(-6)(3)} = -\dfrac{1}{6}$ $\quad (0,-\frac{1}{6})$

y=0: $0 = \dfrac{3}{x^2 - 3x - 18}$ has no solutions.

Additional Points: (–4,0.3), (–2,0.4), (–1,–0.2), (2,–0.15), (4,–0.2), (7,0.3)



21. $y = \dfrac{2x-1}{x^2-4} = \dfrac{2x-1}{(x-2)(x+2)}$

Vertical asymptotes: $x = \pm 2$; horizontal asymptote: $y = 0$ (x–axis).
Intercepts:

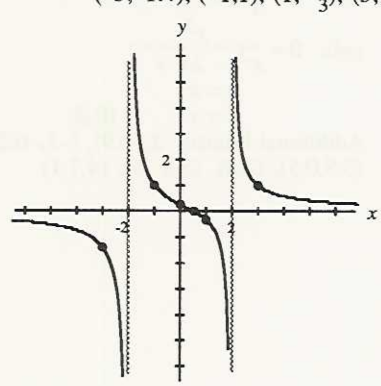
x=0: $y = \frac{-1}{-4} = \frac{1}{4}$ $\quad (0,\frac{1}{4})$

y=0: $0 = \dfrac{2x-1}{x^2-4}$

$0 = 2x - 1$

$\frac{1}{2} = x$ $\quad (\frac{1}{2},0)$

Additional Points:
(–3,–1.4), (–1,1), $(1,-\frac{1}{3})$, (3,1)



25. $y = \dfrac{-2x-3}{x^2-4x} = \dfrac{-2x-3}{x(x-4)}$

Vertical asymptotes:
$x = 0, 4$; horizontal asymptote: $y = 0$ (x–axis).
Intercepts:

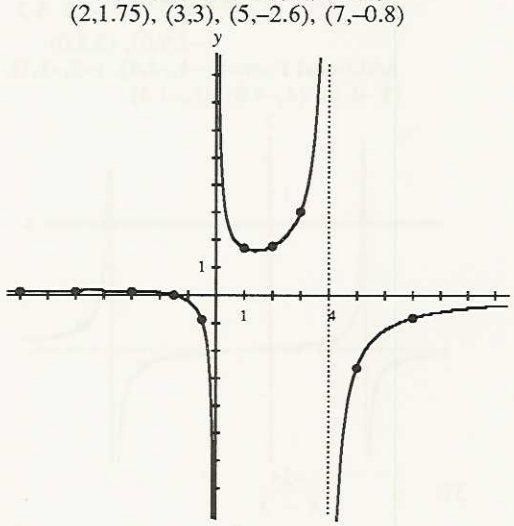
x=0: $y = \dfrac{-3}{0}$ Not defined.

y=0: $0 = \dfrac{-2x-3}{x^2-4x}$

$0 = -2x - 3$

$x = -\frac{3}{2}$ $\quad (-1\frac{1}{2},0)$

Additional Points: (–6,0.15), (–4,0.16), (–2,0.08), (–0.5,–0.9), (1,1.67), (2,1.75), (3,3), (5,–2.6), (7,–0.8)



29. $y = \dfrac{3x-1}{(x^2-2x)(x+1)}$

$= \dfrac{3x-1}{x(x-2)(x+1)}$

Vertical asymptotes: $x = -1, 0, 2$;
horizontal asymptote: $y = 0$ (x–axis).
Intercepts:

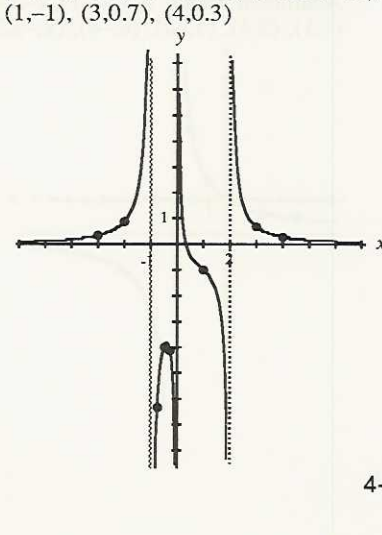
x=0: $y = \frac{-1}{0}$ Not defined

y=0: $0 = \dfrac{3x-1}{(x^2-2x)(x+1)}$

$0 = 3x - 1$

$\frac{1}{3} = x$ $\quad (\frac{1}{3},0)$

Additional Points: (–3,0.3), (–2,0.9), (–0.75,–6.3), (–0.5,–5), (–0.25,–4.1), (1,–1), (3,0.7), (4,0.3)



33. $y = \frac{x}{x^2 - 2x - 15} - 4$

$= \frac{x}{(x-5)(x+3)} - 4$

Vertical asymptotes: $x = -3, 5$

Horizontal asymptote: $y = -4$

Intercepts:

$x{=}0$: $y = 0 - 4 = -4$ $(0,-4)$

$y{=}0$: $0 = \frac{x}{(x-5)(x+3)} - 4$

$4 = \frac{x}{(x-5)(x+3)}$

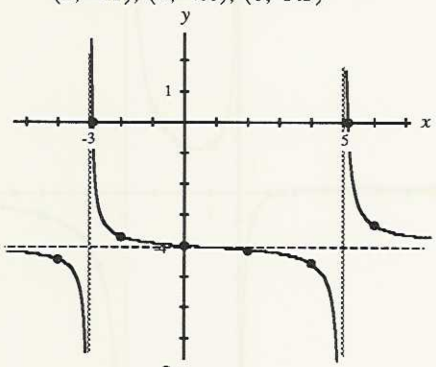
$4(x-5)(x+3) = x$

$4x^2 - 9x - 60 = 0$

$x = \frac{9 \pm \sqrt{1041}}{8} \approx -2.9, 5.2$

$(-2.9,0)$, $(5.2,0)$

Additional Points: $(-4,-4.4)$, $(-2,-3.7)$, $(2,-4.1)$, $(4,-4.6)$, $(6,-3.3)$



37. $y = \frac{-2x}{x-3}$

$= -2 - \frac{6}{x-3}$ Long division

$= \frac{-6}{x-3} - 2$

Graph of $y = \frac{1}{x}$ flipped over, scaled by 6 and shifted.

Vertical asymptote: $x = 3$

Horizontal asymptote: $y = -2$

"Origin": $(3,-2)$

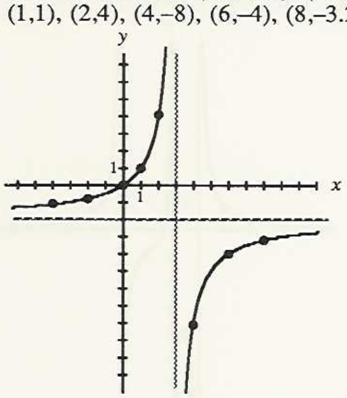
Intercepts:

$x{=}0$: $y = \frac{0}{-2} = 0$ $(0,0)$

$y{=}0$: $0 = \frac{-2x}{x-3}$

$0 = x$ $(0,0)$

Additional Points: $(-4,-1.1)$, $(-2,-0.8)$, $(1,1)$, $(2,4)$, $(4,-8)$, $(6,-4)$, $(8,-3.2)$



41. $y = \frac{-3x^2 + 2x - 1}{x^2 - 4}$

$= -3 + \frac{2x - 13}{(x-2)(x+2)}$

Horizontal asymptote: $y = -3$

Vertical asymptotes: $x = \pm\ 2$

Intercepts:

$x{=}0$: $y = \frac{-1}{-4} = \frac{1}{4}$ $(0, 0.25)$

$y{=}0$: $0 = \frac{-3x^2 + 2x - 1}{x^2 - 4}$

$0 = -3x^2 + 2x - 1$

$0 = 3x^2 - 2x + 1$

$0 = (3x + 1)(x - 1)$

$x = -\frac{1}{3}, 1$ $(-0.3,0)$, $(1,0)$

Additional Points:

$(-5,-4.1)$, $(-3,-6.8)$, $(-1,2)$, $(3,-3.8)$, $(5,-3.1)$, $(9,-2.94)$, $(11,-2.92)$

The value of y at $x = 5$ is less than -3 and at $x = 9$ is more than -3. The coordinate where $y = -3$ can be found be replacing y by -3 and solving.

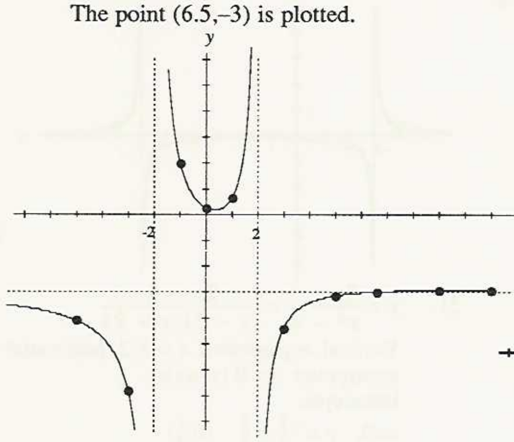
$-3 = \frac{-3x^2 + 2x - 1}{x^2 - 4}$

$-3x^2 + 12 = -3x^2 + 2x - 1$

$13 = 2x$

$6.5 = x.$

The point $(6.5,-3)$ is plotted.



45. $y = \frac{x^3}{x^2 - 2x + 1} = x + 2 + \frac{x - 2}{(x-1)2}$

Slant asymptote: $y = x + 2$

Vertical asymptote: $x = 1$

Intercepts:

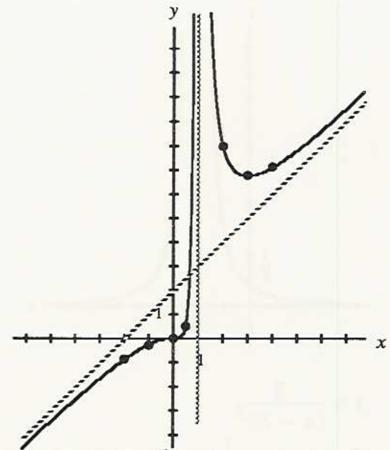
$x{=}0$: $y = \frac{0}{1} = 0$ $(0,0)$

$y{=}0$: $0 = \frac{x^3}{x^2 - 2x + 1}$

$0 = x^3$

$0 = x$ $(0,0)$

Additional Points: $(-2,-0.9)$, $(-1,-0.25)$, $(0.5,0.5)$, $(2,8)$, $(3,6.75)$, $(4,7.1)$



49. $y = \frac{x^3 + 8}{x^2 + 3x - 4}$

$= x - 3 + \frac{13x - 4}{(x+4)(x-1)}$

Slant asymptote: $y = x - 3$

Vertical asymptotes: $x = -4, 1$

Intercepts:

$x{=}0$: $y = \frac{8}{-4} = -2$ $(0,-2)$

$y{=}0$: $0 = \frac{x^3 + 8}{x^2 + 3x - 4}$

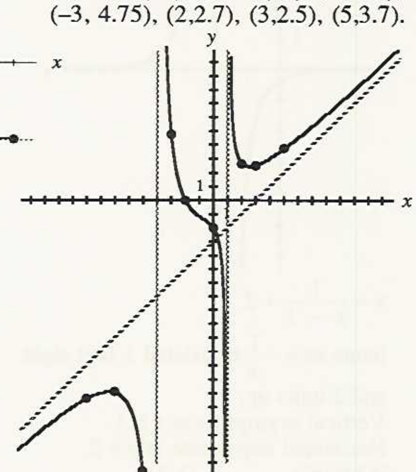
$0 = x^3 + 8$

$-8 = x^3$

$-2 = x$ $(-2,0)$

Additional Points:

$(-9,-14.4)$, $(-7,-14.3)$, $(-5,-20.8)$, $(-3, 4.75)$, $(2,2.7)$, $(3,2.5)$, $(5,3.7)$.



53. $y = \frac{x^2 - 1}{x^2 + 2x - 3} = \frac{(x-1)(x+1)}{(x+3)(x-1)}$

$= \frac{x+1}{x+3} = 1 + \frac{-2}{x+3}, x \neq 1.$

The function f is undefined at $x = 1$ (there is a hole in its graph).

Horizontal Asymptote: $y = 1$.

Vertical asymptote at $x = -3$.

Intercepts:

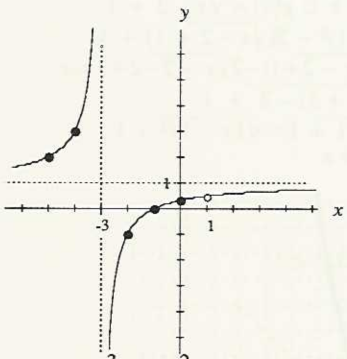
$x{=}0$: $y = \frac{-1}{-3} = \frac{1}{3}$ $(0,\frac{1}{3})$

$y{=}0$: $0 = \frac{x+1}{x+3}$

$0 = x + 1$

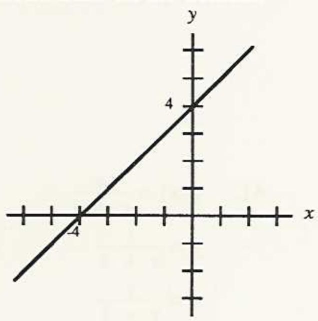
$-1 = x$ $(-1,0)$

Additional Points: $(-5,2)$, $(-4,3)$, $(-2,-1)$



57. $y = \dfrac{x^3 + 4x^2 + 3x + 12}{x^2 + 3}$
$= \dfrac{x^2(x + 4) + 3(x + 4)}{x^2 + 3}$
$= \dfrac{(x + 4)(x^2 + 3)}{x^2 + 3}$
$= x + 4$

This is a straight line with intercepts at (0,4) and (–4,0). Since $x^2 + 3 \neq 0$ for all real values of x there are no restrictions on the domain.



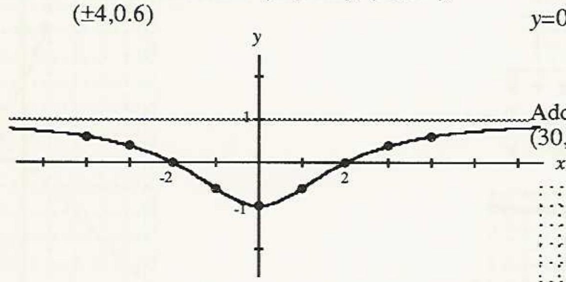
61. $y = \dfrac{x^2 - 4}{x^2 + 4} = 1 + \dfrac{-8}{x^2 + 4}$

No vertical asymptotes.
Horizontal asymptote: $y = 1$.
Intercepts:

x=0: $y = \dfrac{-4}{4} = -1$ (0,–1)

y=0: $0 = \dfrac{x^2 - 4}{x^2 + 4}$
$0 = x^2 - 4$
$x = -2, 2$ (±2,0)

Additional Points: (±1,–0.6), (±3,0.4), (±4,0.6)



65. (a) $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{20}$

$\dfrac{y + x}{xy} = \dfrac{1}{20}$ Add the left members.

$20y + 20x = xy$ Multiply each member by $20xy$

$20x = xy - 20y$ Put all y-terms in the right member

$20x = y(x - 20)$ Factor out y in the right member

$\dfrac{20x}{x - 20} = y$ Divide each member by $x - 20$

(b) Since the degree of the numerator is greater than or equal to the degree of the denominator, divide.

$y = \dfrac{20x}{x - 20}$

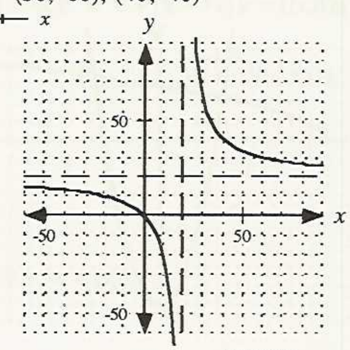
$y = \dfrac{400}{x - 20} + 20$ Long division.

Horizontal asymptote: $y = 20$.
Vertical asymptote: $x = 20$.
Intercepts:

x=0: $y = \dfrac{0}{-20} = 0$ (0,0)

y=0: $0 = \dfrac{20x}{x - 20}$
$0 = 20x$
$0 = x$ (0,0)

Additional Points: (–10,–6.7), (10,20), (30,–60), (40,–40)



69. $f(x) = \dfrac{150000}{x(x + 10)}$

(a) $f(10) = \dfrac{150000}{10(10 + 10)} = \$750$

(b) $f(20) = \dfrac{150000}{20(20 + 10)} = \$250$

(c) $f(30) = \dfrac{150000}{30(30 + 10)} = \$125$

Exercise 4-5

1. $f(x) = 3x - 5$; $g(x) = -2x + 8$
$(3x - 5) + (-2x + 8) = x + 3$
$(3x - 5) - (-2x + 8) = 5x - 13$
$(3x - 5) \cdot (-2x + 8) = -6x^2 + 34x - 40$
$(3x - 5) / (-2x + 8) = \dfrac{3x - 5}{-2x + 8}$
$f[g(x)] = f[(-2x + 8)] = 3(-2x + 8) - 5 = -6x + 19$
$g[f(x)] = g[(3x - 5)] = -2(3x - 5) + 8 = -6x + 18$

5. $f(x) = \dfrac{x - 3}{2x}$; $g(x) = \dfrac{x}{x - 1}$

$\dfrac{x - 3}{2x} + \dfrac{x}{x - 1} = \dfrac{(x - 3)(x - 1) + x(2x)}{2x(x - 1)}$
$= \dfrac{x^2 - 4x + 3 + 2x^2}{2x^2 - 2x} = \dfrac{3x^2 - 4x + 3}{2x^2 - 2x}$

$\dfrac{x - 3}{2x} - \dfrac{x}{x - 1} = \dfrac{(x - 3)(x - 1) - x(2x)}{2x(x - 1)}$
$= \dfrac{x^2 - 4x + 3 - 2x^2}{2x^2 - 2x} = \dfrac{-x^2 - 4x + 3}{2x^2 - 2x}$

$\dfrac{x - 3}{2x} \cdot \dfrac{x}{x - 1} = \dfrac{x - 3}{2} \cdot \dfrac{1}{x - 1} = \dfrac{x - 3}{2x - 2}$

$\dfrac{x - 3}{2x} / \dfrac{x}{x - 1} = \dfrac{x - 3}{2x} \cdot \dfrac{x - 1}{x} = \dfrac{x^2 - 4x + 3}{2x3}$

$f[g(x)] = f[(\dfrac{x}{x - 1})] = \dfrac{\dfrac{x}{x - 1} - 3}{2 \cdot \dfrac{x}{x - 1}} \cdot \dfrac{x - 1}{x - 1}$

$= \dfrac{x - 3(x - 1)}{2x} = \dfrac{-2x + 3}{2x}$

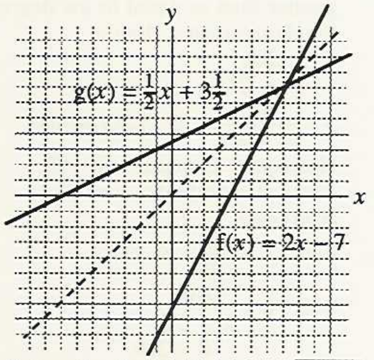
$g[f(x)] = g[(\dfrac{x - 3}{2x})] = \dfrac{\dfrac{x - 3}{2x}}{\dfrac{x - 3}{2x} - 1} \cdot \dfrac{2x}{2x}$

$= \dfrac{x - 3}{(x - 3) - 2x} = \dfrac{x - 3}{-x - 3} = -\dfrac{x - 3}{x + 3}$

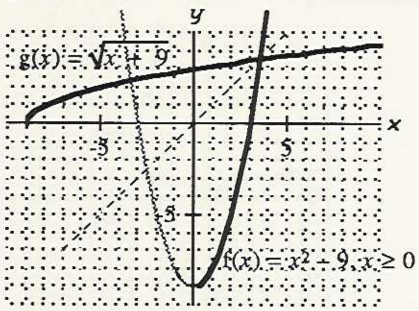
9. $f(x) = x$; $g(x) = 3$
$(x) + (3) = x + 3$
$(x) - (3) = x - 3$
$(x) \cdot (3) = 3x$
$(x) / (3) = \frac{x}{3}$
$f[g(x)] = f[3] = 3$
$g[f(x)] = g[x] = 3$

13. $f(x) = 2x - 7$; $g(x) = \frac{1}{2}x + 3\frac{1}{2}$
$f(g(x)) = 2(g(x)) - 7 = 2(\frac{1}{2}x + 3\frac{1}{2}) - 7$
$= x + 7 - 7 = x$

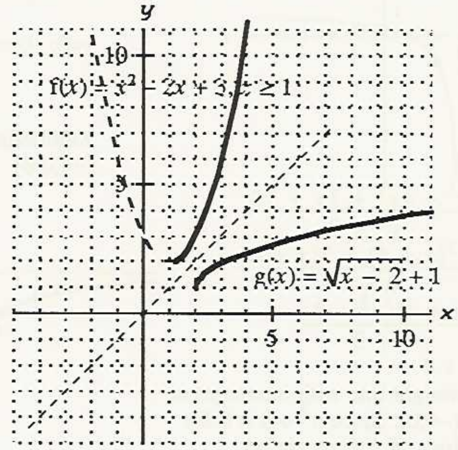
$g(f(x)) = \frac{1}{2}(2x - 7) + 3\frac{1}{2} = x - \frac{7}{2} + 3\frac{1}{2} = x$



17. $f(x) = x^2 - 9, x \geq 0$; $g(x) = \sqrt{x + 9}$
$f(g(x)) = (\sqrt{x+9})^2 - 9 = x + 9 - 9 = x$
$g(f(x)) = \sqrt{(x^2 - 9) + 9} = \sqrt{x^2} = x$



21. $f(x) = x^2 - 2x + 3, x \geq 1$; $g(x) = \sqrt{x - 2} + 1$
$f(g(x)) = (\sqrt{x-2} + 1)^2 - 2(\sqrt{x-2} + 1) + 3$
$= ((x-2) + 2\sqrt{x-2} + 1) - 2\sqrt{x-2} - 2 + 3 = x$
$g(f(x)) = \sqrt{(x^2 - 2x + 3) - 2} + 1$
$= \sqrt{x^2 - 2x + 1} + 1 = \sqrt{(x-1)^2} + 1$
$= (x - 1) + 1 = x$



25. $f(x) = 4x - 5$
$y = 4x - 5$
$x = 4y - 5$
$x + 5 = 4y$
$\frac{x + 5}{4} = y$
$f^{-1}(x) = \frac{1}{4}x + \frac{5}{4}$

29. $g(x) = x^2 - 9, x \geq 0$
$y = x^2 - 9, x \geq 0$
$x = y^2 - 9, y \geq 0$
$x + 9 = y^2, y \geq 0$
$\pm\sqrt{x+9} = y, y \geq 0$
$y = +\sqrt{x + 9}$
$g^{-1}(x) = \sqrt{x + 9}$

33. $h(x) = \sqrt{x - 4}$
$y = \sqrt{x - 4}$
$x = \sqrt{y - 4}$
$x^2 = y - 4$
$y = x^2 + 4$; This is not a one-to-one function.
Observe that $y = \sqrt{x - 4} \geq 0$, so we know that for h, $y \geq 0$.
Thus, for h^{-1} $x \geq 0$.
$h^{-1}(x) = x^2 + 4, x \geq 0$

37. $f(x) = \sqrt[3]{4x - 5}$
$y = \sqrt[3]{4x - 5}$
$x = \sqrt[3]{4y - 5}$
$x^3 = 4y - 5$
$\frac{x^3 + 5}{4} = y$
$f^{-1}(x) = \frac{x^3 + 5}{4}$

41. $g(x) = \frac{x}{x + 1}$
$y = \frac{x}{x + 1}$
$x = \frac{y}{y + 1}$
$xy + x = y$
$xy - y = -x$
$y(x - 1) = -x$
$y = \frac{-x}{x - 1}$
$g^{-1}(x) = \frac{-x}{x - 1}$

45. $h(x) = x^2 - 2x - 9, x \geq 1$
$y = x^2 - 2x - 9, y \geq 1$
$x = y^2 - 2y - 9, y \geq 1$
$0 = y^2 - 2y + (-x - 9)$; using the Quadratic Formula,
$y = \frac{-(-2) \pm\sqrt{(-2)^2 - 4(1)(-x - 9)}}{2(1)}$
$y = \frac{2 \pm \sqrt{4(x + 10)}}{2} = \frac{2 \pm 2\sqrt{x + 10}}{2}$
$y = 1 \pm \sqrt{x + 10},\ y \geq 1$
For $y \geq 1$ we need to add values to 1:
$h^{-1}(x) = 1 + \sqrt{x + 10}$

49. $C(x) = (P \circ V)(x)$
$= P(V(x))$
$= \frac{1}{2}[V(x)]$
$= \frac{1}{2}[x^3]$
$= \frac{x^3}{2}$

53. $A(t) = (d_h \bullet d_v)(t)$
$= d_h(t) \bullet d_v(t)$
$= (5t)(16t^2)$
$= 80t^3$

57. $R(x) = \frac{20x}{20 + x}$
$y = \frac{20x}{20 + x}$
$x = \frac{20y}{20 + y}$
$20x + xy = 20y$
$20x = 20y - xy$
$20x = y(20 - x)$
$y = \frac{20x}{20 - x}$
$R^{-1}(x) = \frac{20x}{20 - x}$

61. $f(x) = ax + b$
$y = ax + b$
$x = ay + b$
$x - b = ay$
$\frac{x - b}{a} = y$
$f^{-1}(x) = \frac{1}{a}x - \frac{b}{a}$
The inverse does not exist if $a = 0$.

Exercise 4-6

1. $\frac{x - 10}{x^2 - 5x + 4} = \frac{x - 10}{(x - 4)(x - 1)}$
$\frac{x - 10}{(x - 4)(x - 1)} = \frac{A}{x - 4} + \frac{B}{x - 1}$
$\frac{x - 10}{(x - 4)(x - 1)} \bullet (x-4)(x-1)$
$= \frac{A}{x - 4} \bullet (x - 4)(x - 1) + \frac{B}{x - 1} \bullet (x-4)(x-1)$
$\boldsymbol{x - 10 = A(x - 1) + B(x - 4)}$
Let $x = 1$: $-9 = A(0) + B(-3)$

$3 = B$

Let $x = 4$: $-6 = A(3) + B(0)$

$-2 = A$

Thus, $\dfrac{x-10}{x^2-5x+4} = \dfrac{-2}{x-4} + \dfrac{3}{x-1}$.

5. $\dfrac{4x^3-6x^2-1}{2x^2-3x+1} = 2x + \dfrac{-2x-1}{(2x-1)(x-1)}$

$\dfrac{-2x-1}{(2x-1)(x-1)} = \dfrac{A}{2x-1} + \dfrac{B}{x-1}$

$\dfrac{-2x-1}{(2x-1)(x-1)}\cdot(2x-1)(x-1)$

$= \dfrac{A}{2x-1}\cdot(2x-1)(x-1) + \dfrac{B}{x-1}\cdot(2x-1)(x-1)$

$\mathbf{-2x-1 = A(x-1) + B(2x-1)}$

Let $x = 1$: $-3 = A(0) + B(1)$

$-3 = B$

Let $x = \frac{1}{2}$: $-2 = A(-\frac{1}{2}) + B(0)$

$4 = A$

Thus, $\dfrac{-2x-1}{(2x-1)(x-1)} = \dfrac{4}{2x-1} + \dfrac{-3}{x-1}$, and

$\dfrac{4x^3-6x^2-1}{2x^2-3x+1} = 2x + \dfrac{4}{2x-1} + \dfrac{-3}{x-1}$.

9. $\dfrac{3x^2-4x-1}{(x-1)^2(x-2)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2}$

$\dfrac{3x^2-4x-1}{(x-1)^2(x-2)}\cdot(x-1)^2(x-2) = \dfrac{A}{x-1}\cdot(x-1)^2(x-2) + \dfrac{B}{(x-1)^2}\cdot(x-1)^2(x-2) + \dfrac{C}{x-2}\cdot(x-1)^2(x-2)$

$\mathbf{3x^2-4x-1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2}$

Let $x = 1$: $-2 = A(0) + B(-1) + C(0)$

$2 = B$

Let $x = 2$: $3 = A(0) + B(0) + C(1)$

$3 = C$

Let $x = 0$: $-1 = 2A - 2(2) + 3$ $\quad B = 2, C = 3.$

$0 = A$

Thus, $\dfrac{3x^2-4x-1}{(x-1)^2(x-2)} = \dfrac{2}{(x-1)^2} + \dfrac{3}{x-2}$.

13. $\dfrac{3x^3-11x^2+x-17}{(x-3)^2(x+1)^2} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2}$

$\dfrac{3x^3-11x^2+x-17}{(x-3)^2(x+1)^2}\cdot(x-3)^2(x+1)^2 = \dfrac{A}{x-3}\cdot(x-3)^2(x+1)^2 + \dfrac{B}{(x-3)^2}\cdot(x-3)^2(x+1)^2 +$

$\dfrac{C}{x+1}\cdot(x-3)^2(x+1)^2 + \dfrac{D}{(x+1)^2}\cdot(x-3)^2(x+1)^2$

$\mathbf{3x^3-11x^2+x-17 = A(x-3)(x+1)^2 + B(x+1)^2 + C(x-3)^2(x+1) + D(x-3)^2}$

Let $x = 3$: $-32 = A(0) + B(16) + C(0) + D(0)$

$-2 = B$

Let $x = -1$: $-32 = A(0) + B(0) + C(0) + D(16)$

$-2 = D$

We now make any other two choices for x.

Let $x = 0$: $-17 = -3A + (-2) + 9C + 9(-2)$ $\quad B = -2, D = -2.$

$-17 = -3A - 2 + 9C - 18$

$3 = -3A + 9C$

[1] $1 = -A + 3C$

Let $x = 1$: $-24 = -8A + 4(-2) + 8C + 4(-2)$

$-24 = -8A - 8 + 8C - 8$ $\quad B = -2, D = -2.$

$-8 = -8A + 8C$

[2] $1 = A - C$

By [1], $A = 3C - 1$; plugging this into [2] we obtain

$1 = (3C - 1) - C$

$1 = 2C - 1$

$2 = 2C$

$C = 1.$

Since $A = 3C - 1$, $A = 3 - 1 = 2$.

Thus, $\dfrac{3x^3-11x^2+x-17}{(x-3)^2(x+1)^2} = \dfrac{2}{x-3} + \dfrac{-2}{(x-3)^2} + \dfrac{1}{x+1} + \dfrac{-2}{(x+1)^2}$.

17. $\dfrac{x^2+4x+4}{(x-1)(x^2+x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

$\dfrac{x^2+4x+4}{(x-1)(x^2+x+1)}\cdot(x-1)(x^2+x+1) = \dfrac{A}{x-1}\cdot(x-1)(x^2+x+1) + \dfrac{Bx+C}{x^2+x+1}\cdot(x-1)(x^2+x+1)$

$\mathbf{x^2+4x+4 = A(x^2+x+1) + (Bx+C)(x-1)}$

Let $x = 1$: $9 = A(3) + B(0)$

$3 = A$

Let $x = 0$: $4 = 3(1) + C(-1)$ $\quad A = 3.$

$-1 = C$

Let $x = -1$: $1 = 3(1) + (-B - 1)(-2)$ $\quad A = 3, C = -1.$

$1 = 3 + 2B + 2$

$-2 = B$

Thus, $\dfrac{x^2+4x+4}{(x-1)(x^2+x+1)} = \dfrac{3}{x-1} + \dfrac{-2x-1}{x^2+x+1}$.

21. $\frac{x^2 - 11x - 2}{(x-3)(x^2+x+1)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+x+1}$

$\frac{x^2 - 11x - 2}{(x-3)(x^2+x+1)} \cdot (x-3)(x^2+x+1) = \frac{A}{x-3} \cdot (x-3)(x^2+x+1) + \frac{Bx+C}{x^2+x+1} \cdot (x-3)(x^2+x+1)$

$\mathbf{x^2 - 11x - 2 = A(x^2 + x + 1) + (Bx + C)(x - 3)}$

Let $x = 3$: $-26 = A(13)$
$-2 = A$

Let $x = 0$: $-2 = -2(1) + C(-3)$ $A = -2.$
$C = 0$

Let $x = 1$: $-12 = -2(3) + B(-2)$ $A = -2, C = 0.$
$B = 3$

$\frac{x^2 - 11x - 2}{(x-3)(x^2+x+1)} = \frac{3x}{x^2+x+1} - \frac{2}{x-3}$

25. $\frac{2x+15}{x^2+15x+50} = \frac{2x+15}{(x+5)(x+10)} = \frac{A}{x+5} + \frac{B}{x+10}$

$\frac{2x+15}{(x+5)(x+10)} \cdot (x+5)(x+10) = \frac{A}{x+5} \cdot (x+5)(x+10) + \frac{B}{x+10} \cdot (x+5)(x+10)$

$\mathbf{2x + 15 = A(x + 10) + B(x + 5)}$

Let $x = -5$: $5 = A(5)$; $A = 1$

Let $x = -10$: $-5 = B(-5)$; $B = 1$

$\frac{2x+15}{(x+5)(x+10)} = \frac{1}{x+5} + \frac{1}{x+10}$

29. $\frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$

Thus,

$\frac{2}{1\cdot3} + \frac{2}{2\cdot4} + \frac{2}{3\cdot5} + \frac{2}{4\cdot6} + \ldots + \frac{2}{97\cdot99} + \frac{2}{98\cdot100} + \frac{2}{99\cdot101}$

$= \frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \ldots + \frac{1}{97} - \frac{1}{99} + \frac{1}{98} - \frac{1}{100} + \frac{1}{99} - \frac{1}{101}$

$= \frac{1}{1} + \frac{1}{2} - \frac{1}{100} - \frac{1}{101}$

$= \frac{3}{2} - (\frac{1}{100} + \frac{1}{101})$

$= \frac{3}{2} - \frac{201}{10100}$

$= \frac{3(10100) - 2(201)}{2(10100)}$

$= \frac{29898}{20200} = \frac{14949}{10100} \approx 1.4801$

Chapter 4 Review

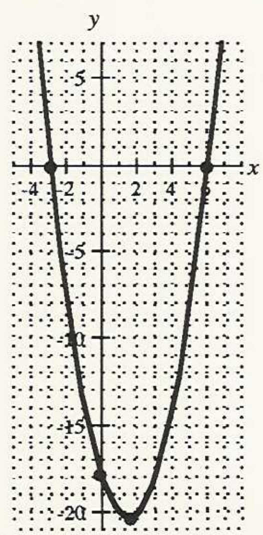
1. $y = x^2 - 3x - 18$

$y = x^2 - 3x + \frac{9}{4} - 18 - \frac{9}{4}$

$y = (x - \frac{3}{2})^2 - \frac{81}{4}$ Vertex: $(1\frac{1}{2}, -20\frac{1}{4})$

x-intercept (y=0): $0 = x^2 - 3x - 18$
$0 = (x-6)(x+3)$
$x = -3$ or 6
$(-3,0)$, $(6,0)$

y-intercept (x=0): $y = -18$ $(0,-18)$



3. $y = x^2 - 4x$

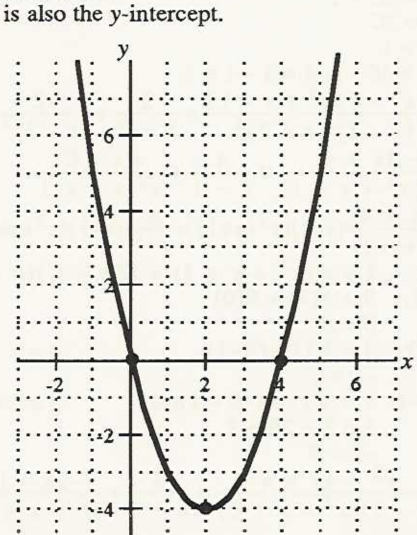
$y = x^2 - 4x + 4 - 4$

$y = (x-2)^2 - 4$

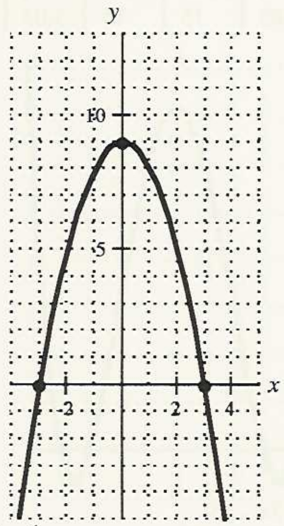
Vertex: $(2,-4)$

x-intercept (y=0):
$0 = x^2 - 4x$
$0 = x(x-4)$
$x = 0$ or 4
$(0,0)$, $(4,0)$

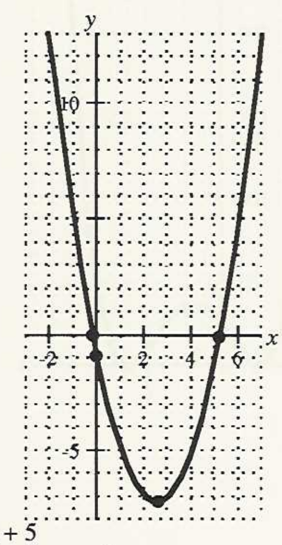
$(0,0)$ is also the y-intercept.



5. $y = 9 - x^2$
$y = -x^2 + 9$ Vertex: (0,9)
x-intercept (y=0): $0 = 9 - x^2$
$0 = (3 - x)(3 + x)$
$x = \pm 3$
(–3,0), (3,0)
y-intercept (x=0): $y = 9$ (0,9)

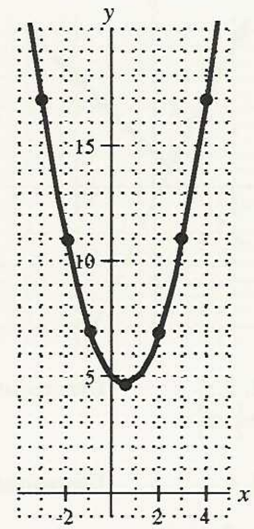


7. $y = x^2 - 5x - 1$
$y = x^2 - 5x + \frac{25}{4} - 1 - \frac{25}{4}$
$y = (x - \frac{5}{2})^2 - \frac{29}{4}$ Vertex: $(2\frac{1}{2}, -7\frac{1}{4})$
x-intercept (y=0): $0 = (x - \frac{5}{2})^2 - \frac{29}{4}$
$\frac{29}{4} = (x - \frac{5}{2})^2$
$\pm\frac{\sqrt{29}}{2} = x - \frac{5}{2}$
$x = \frac{5}{2} \pm \frac{\sqrt{29}}{2} \approx 5.2, -0.2$
(–0.2,0), (5.2,0)
y-intercept (x=0): $y = -1$ (0,–1)

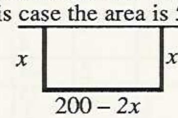


9. $y = x^2 - x + 5$
$y = x^2 - x + \frac{1}{4} + 5 - \frac{1}{4}$
$y = (x - \frac{1}{2})^2 + 4\frac{3}{4}$ Vertex: $(\frac{1}{2}, 4\frac{3}{4})$
x-intercept (y=0): $0 = (x - \frac{1}{2})^2 + 4\frac{3}{4}$
$-4\frac{3}{4} = (x - \frac{1}{2})^2$
No real solution, so no x-intercepts.

y-intercept (x=0): $y = 5$ (0,5)
Additional Points: (–3,17), (–2,11), (–1,7), (2,7), (3,11), (4,17)



11. The area, A, is $x(200 - 2x)$. We find the vertex for the parabola $A = x(200 - 2x)$.
$A = 200x - 2x^2$
$A = -2(x^2 - 100x)$
$A = -2(x^2 - 100x + 50^2) + 2(50^2)$
$A = -2(x - 50)^2 + 5000$
Vertex: $(x, A) = (50, 5000)$.
Thus the area is maximized if $x = 50$, so the dimensions are 50 and 100; in this case the area is 5000 sq. ft.



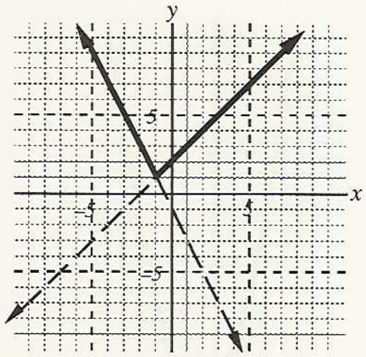
13. The last problem showed that the maximum area for a rectangle with a perimeter of 400 feet is 10,000 sq. ft. Now we find the area of the half-circle.

The circumference (perimeter) of a circle is $C = 2\pi r$, so half that is πr, where r is the radius. If x is the radius, then there are $400 - 2x$ feet left for this circular part. Thus:
$\pi r = 400 - 2x$
$r = \frac{400 - 2x}{\pi}$
We also know that $r = x$. Therefore, substituting x for r we obtain
$x = \frac{400 - 2x}{\pi}$
$\pi x = 400 - 2x$
$\pi x + 2x = 400$
$x(\pi + 2) = 400$
$x = \frac{400}{\pi + 2} \approx 77.8$ feet.
Thus the radius of the circle is $\frac{400}{\pi + 2}$ feet. The area of a circle is πr^2, so the area of half a circle is $\frac{\pi}{2} r^2 = \frac{\pi}{2}\left(\frac{400}{\pi + 2}\right)^2$ $\approx 9{,}507$ sq ft.

Thus, the rectangle (square) will give a larger area for a given perimeter.

15. $h(x) = \begin{cases} -2x - 1, & x < -1 \\ x + 2, & x \geq -1 \end{cases}$

Graph the two lines $y = -2x - 1$ and $y = x + 2$.



17. $2x^4 - 3x^2 + 6$

Numerator: 1, 2, 3, 6 Denominator: 1, 2

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

19. $x^5 - x^3 - 4$

Numerator: 1, 2, 4 Denominator: 1

$\pm1, \pm2, \pm4$

21. $f(x) = 2x^4 - 5x^3 + 2x^2 - 1$

	2	−5	2	0	−1
		6	3	15	45
3	2	1	5	15	44

$f(x) \div (x - 3)$

$= 2x^3 + x^2 + 5x + 15 + \frac{44}{x - 3}$, and $f(3) = 44$.

23. $f(x) = \frac{1}{2}x^3 - 3x^2 + \frac{3}{4}x - 3$

	$\frac{1}{2}$	−3	$\frac{3}{4}$	−3
		2	−4	−13
4	$\frac{1}{2}$	−1	$-\frac{13}{4}$	−16

$f(x) \div (x - 4) = \frac{1}{2}x^2 - x - \frac{13}{4} - \frac{16}{x - 4}$ and $f(4) = -16$.

25. $g(x) = 2x^4 - x^3 - 9x^2 + 4x + 4$

(a) $f(x)$:

2 sign changes, so 0 or 2 positive real zeros.

$f(-x) = 2x^4 + x^3 - 9x^2 - 4x + 4$

2 sign changes; 0 or 2 negative real zeros.

(b) 4: 1,2,4 ; 2: 1,2

Possible rational zeros: $\pm(1, 2, 4, \frac{1}{2})$

(c,d) Using synthetic division, 1 and 2 are zeros, giving

$f(x) = (x - 1)(x - 2)(2x^2 + 5x + 2)$
$= (x - 1)(x - 2)(x + 2)(2x + 1)$,

so all zeros are $1, 2, -2, -\frac{1}{2}$.

27. $f(x) = 16x^5 - 48x^4 - 40x^3 + 120x^2 + 9x - 27$

(a) $f(x)$: 3 sign changes, so there are 1 or 3 positive real zeros.

$f(-x) = -16x^5 - 48x^4 + 40x^3 + 120x^2 - 9x - 27$

2 sign changes, so there are 0 or 2 positive real zeros.

(b) 27: 1, 3, 9, 27 16: 1, 2, 4, 8, 16

Possible rational zeros: $\pm(1, 3, 9, 27, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \frac{1}{4}, \frac{3}{4}, \frac{9}{4}, \frac{27}{4}, \frac{1}{8}, \frac{3}{8}, \frac{9}{8}, \frac{27}{8}, \frac{1}{16}, \frac{3}{16}, \frac{9}{16}, \frac{27}{16})$

(c,d) Synthetic division shows that 3 is a zero, giving

$f(x) = (x - 3)(16x^4 - 40x^2 + 9)$
$= (x - 3)(4x^2 - 9)(4x^2 - 1)$
$= (x - 3)(2x - 3)(2x + 3)(2x - 1)(2x + 1)$

so all the zeros for f are $3, \pm\frac{3}{2}, \pm\frac{1}{2}$.

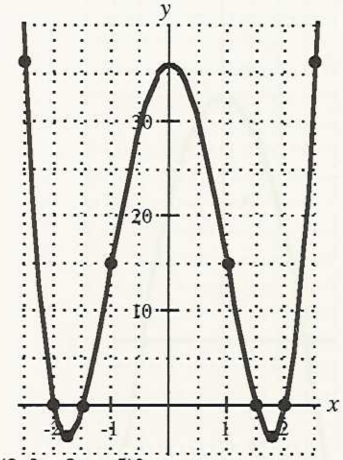
29. $h(x) = (x^2 - 4)(4x^2 - 9)$
$= (x - 2)(x + 2)(2x - 3)(2x + 3)$

x-intercepts: $\pm2, \pm 1\frac{1}{2}$

y-intercept: $h(0) = 36$

Additional Points:

x	$-2\frac{1}{2}$	$-1\frac{3}{4}$	−1	1	$1\frac{3}{4}$	$2\frac{1}{2}$
y	36	−3.05	15	15	−3.05	36



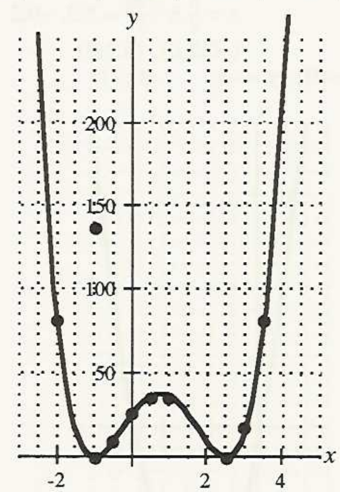
31. $f(x) = (2x^2 - 3x - 5)^2$
$= [(2x - 5)(x + 1)]^2$
$= (2x - 5)^2(x + 1)^2$

x-intercepts: $2\frac{1}{2}, -1$; both zeros have multiplicity 2, so the graph does not cross the axis at the zeros. In fact it can be seen from its definition that $f(x) \geq 0$ for all x.

y-intercept: $f(0) = 25$

Additional Points:

x	−2	−0.5	0.5	1	3	3.5
y	81	9	36	36	16	81



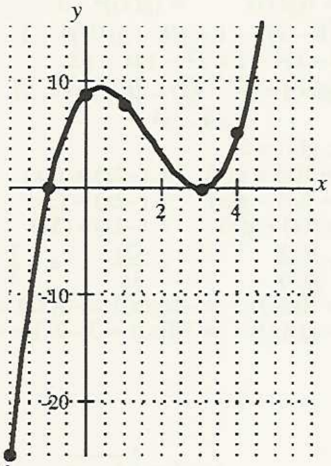
33. $f(x) = (x-3)^2(x+1)$

x-intercepts at -1, and at 3 (multiplicity 2, so the graph does not cross at 3).

y-intercept: $f(0) = 9$

Additional Points:

x	-2	1	2	4
y	-25	8	3	5



35. $f(x) = \dfrac{2}{(x-3)^3}$

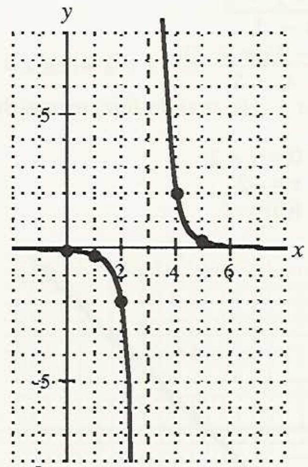
Like $y = \dfrac{1}{x^3}$, which is like $y = \dfrac{1}{x}$, but shifted right 3 units, and with a vertical scaling factor of 2.

Vertical asymptote at $x = 3$.

y-intercept: $f(0) = -\dfrac{2}{27}$.

Additional Points:

x	1	2	4	5
y	$-\frac{1}{4}$	-2	2	$\frac{1}{4}$



37. $f(x) = \dfrac{3}{x^2 - 4x - 45}$

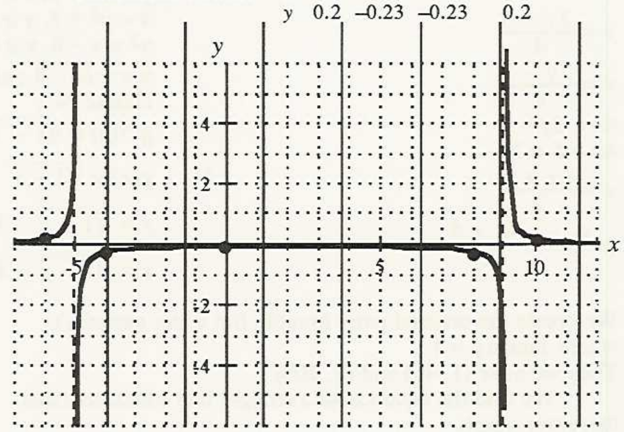
$= \dfrac{3}{(x-9)(x+5)}$

Vertical asymptotes at -5, 9.

y-intercept: $f(0) = -\frac{1}{15}$

Additional Points:

x	-6	-4	8	10
y	0.2	-0.23	-0.23	0.2



39. $g(x) = \dfrac{x^3}{(x-1)(x^2-9)}$

Since the degree of the numerator is greater than or equal to the degree of the denominator, we use long division.

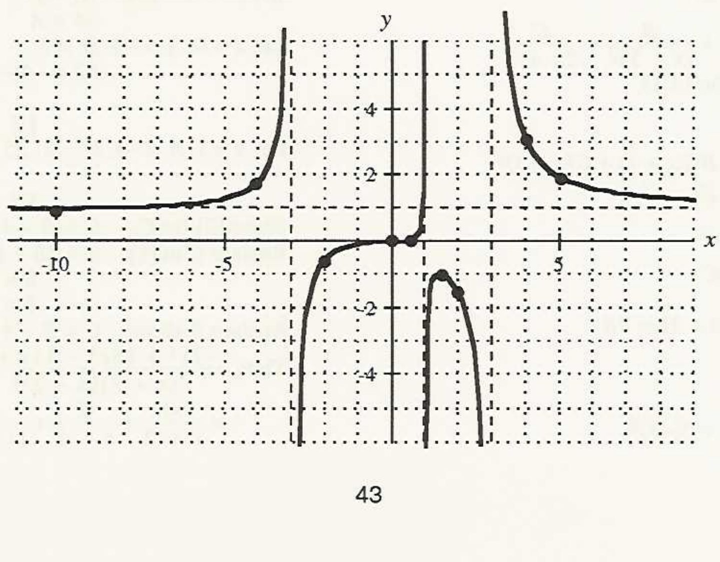
$$\frac{x^3}{(x-1)(x^2-9)} = \frac{x^3}{x^3 - x^2 - 9x + 9} = 1 + \frac{x^2 + 9x - 9}{(x-1)(x^2-9)}.$$

Thus there is a horizontal asymptote at $y = 1$, and vertical asymptotes at -3, 1, 3.

y-intercept: $g(0) = 0$

Additional Points:

x	-10	-4	-2	0.5	1.5	2	4	5
y	0.999	1.8	0.53	0.03	-1	-1.6	3.04	1.95



41. $f(x) = \dfrac{x^2 - x - 6}{x - 3}$

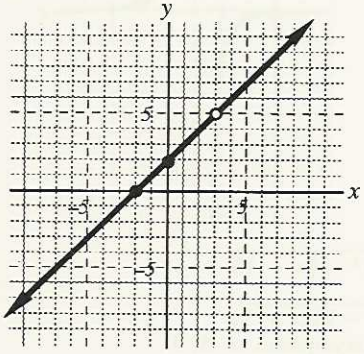
$= \dfrac{(x-3)(x+2)}{x-3} = x + 2$ when $x \neq 3$.

Thus $f(x) = x + 2$ (a straight line) except that it is not defined at $x = 3$.

x-intercept: $0 = x + 2$

$x = -2$

y-intercept: $f(0) = 2$



43. $f(x) = x^4 - 1$, $g(x) = \sqrt{8 - x}$

$(f + g)(x) = f(x) + g(x) \quad = x^4 - 1 + \sqrt{8 - x}$

$(f - g)(x) = f(x) - g(x) \quad = x^4 - 1 - \sqrt{8 - x}$

$(f \cdot g)(x) = f(x) \cdot g(x) \quad = (x^4 - 1)(\sqrt{8 - x})$

$(f / g)(x) = \dfrac{f(x)}{g(x)} \quad = \dfrac{x^4 - 1}{\sqrt{8 - x}}$

$(f \circ g)(x) = f(g(x)) \quad = (g(x))^4 - 1$

$= (\sqrt{8-x})^4 - 1 = ((8 - x)^{1/2})^4 - 1$

$= (8 - x)^2 - 1 = x^2 - 16x + 63$

$(g \circ f)(x) = g(f(x)) = \sqrt{8 - f(x)} = \sqrt{8 - (x^4 - 1)}$

$= \sqrt{9 - x^4}$

45. $f(x) = -2x$; $g(x) = 3x$

$(f + g)(x) = f(x) + g(x) \quad = (-2x) + (3x) \quad = x$

$(f - g)(x) = f(x) - g(x) \quad = (-2x) - (3x) \quad = -5x$

$(f \cdot g)(x) = f(x) \cdot g(x) \quad = (-2x) \cdot (3x) \quad = -6x^2$

$(f / g)(x) = \dfrac{f(x)}{g(x)} \quad = \dfrac{-2x}{3x} \quad = -\dfrac{2}{3}$

$(f \circ g)(x) = f(g(x)) \quad = -2(g(x)) = -2(3x) = -6x$

$(g \circ f)(x) = g(f(x)) \quad = 3(f(x)) = 3(-2x) = -6x$

47. $g(x) = \dfrac{2x - 5}{4}$

$y = \dfrac{2x - 5}{4}$

$x = \dfrac{2y - 5}{4}$

$4x = 2y - 5$

$4x + 5 = 2y$

$y = \dfrac{4x + 5}{2}$

$g^{-1}(x) = \dfrac{4x + 5}{2}$

49. $g(x) = x^2 + 8, x \geq 0$

$y = x^2 + 8, x \geq 0$

$x = y^2 + 8, y \geq 0$

$y^2 = x - 8, y \geq 0$

$y = \pm\sqrt{x - 8}$; since $y \geq 0$ in g^{-1}, choose "+".

$g^{-1}(x) = \sqrt{x - 8}$

51. $g(x) = \sqrt[3]{1 - x} - 3$

$y = \sqrt[3]{1 - x} - 3$

$x = \sqrt[3]{1 - y} - 3$

$x + 3 = \sqrt[3]{1 - y}$

$(x + 3)^3 = 1 - y$

$y = 1 - (x^3 + 9x^2 + 27x + 27)$

$g^{-1}(x) = -x^3 - 9x^2 - 27x - 26$

53. We create the ordered pairs (month, hot water capacity), where January = 1.

Thus we have (1, 40) and (8, 200).

We find the equation of a straight line which contains these two points.

$m = \dfrac{200 - 40}{8 - 1} = \dfrac{160}{7}$

$y - 40 = \dfrac{160}{7}(x - 1)$

$y = \dfrac{160}{7}x - \dfrac{160}{7} + 40$

$f(x) = \dfrac{160}{7}x + \dfrac{120}{7}$ is the required function.

To predict the capacity in June (month 6) compute f(6) $= \dfrac{160}{7}(6) + \dfrac{120}{7} \approx 154$ gallons.

55. $\dfrac{13x^2 - 52x + 32}{(x - 3)^2(2x + 1)} = \dfrac{A}{x - 3} + \dfrac{B}{(x - 3)^2} + \dfrac{C}{2x + 1}$

Multiply each member by the LCD, $(x - 3)^2(2x + 1)$:

$13x^2 - 52x + 32$

$= A(x - 3)(2x + 1) + B(2x + 1) + C(x - 3)^2$

Let $x = -\frac{1}{2}$: $\frac{13}{4} + 26 + 32 = C(-\frac{7}{2})^2$

$\frac{245}{4} = \frac{49}{4}C$

$245 = 49C$

$C = 5$

Let $x = 3$: $\quad 117 - 156 + 32 = 7B$

$-7 = 7B$

$-1 = B$

Let $x = 0$, $C = 5$, $B = -1$:

$32 = A(-3) + (-1)(1) + 5(-3)^2$

$32 = -3A + 44$

$-12 = -3A$

$4 = A$

Thus $\dfrac{13x^2 - 52x + 32}{(x - 3)^2(2x + 1)}$

$= \dfrac{4}{x - 3} + \dfrac{-1}{(x - 3)^2} + \dfrac{5}{2x + 1}$

57. $\dfrac{-2x^3 + 18x^2 - 53x + 62}{(x + 2)(x - 2)^3}$

$= \dfrac{A}{x + 2} + \dfrac{B}{x - 2} + \dfrac{C}{(x - 2)^2} + \dfrac{D}{(x - 2)^3}$

$-2x^3 + 18x^2 - 53x + 62 = A(x - 2)^3 + B(x + 2)(x - 2)^2 + C(x + 2)(x - 2) + D(x + 2)$

Let $x = 2$: $\quad 12 = 4D$

$3 = D$

Let $x = -2$: $256 = A(-4)^3$

$-4 = A$

Let $x = 0$, $A = -4$, $D = 3$:

$62 = -4(-8) + B(2)(4) + C(-4) + 3(2)$

$24 = 8B - 4C$

[1] $\quad 6 = 2B - C$

Let $x = 1$, $A = -4$, $D = 3$: $25 = -4(-1) + B(3) + C(-3) + 3(3)$

$12 = 3B - 3C$

[2] $\quad 4 = B - C$

Solve [2] for C: $\quad C = B - 4$

Replace C in [1]: $\quad 6 = 2B - (B - 4)$

$6 = B + 4$

$2 = B$

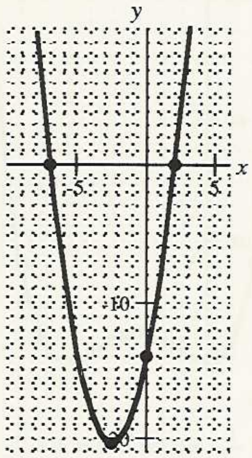
Replace B above: $\quad C = B - 4 = 2 - 4 = -2$

Thus, $\dfrac{-2x^3 + 18x^2 - 53x + 62}{(x + 2)(x - 2)^3}$

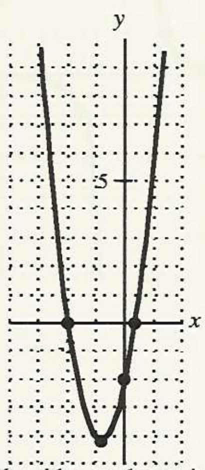
$= \dfrac{-4}{x + 2} + \dfrac{2}{x - 2} + \dfrac{-2}{(x - 2)^2} + \dfrac{3}{(x - 2)^3}$.

Chapter 4 Test

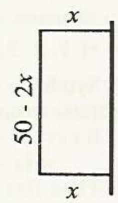
1. $y = x^2 + 5x - 14$
   $y = x^2 + 5x + \frac{25}{4} - 14 - \frac{25}{4}$
   $y = (x + \frac{5}{2})^2 - \frac{81}{4}$ Vertex $(-2\frac{1}{2}, -20\frac{1}{4})$
   x-intercept (y=0): $0 = x^2 + 5x - 14$
   $0 = (x + 7)(x - 2)$
   $x = -7$ or 2
   $(-7,0)$, $(2,0)$
   y-intercept (x=0): $y = -14$ $(0,-14)$



3. $y = 3x^2 + 5x - 2$
   $y = 3(x^2 + \frac{5}{3}x) - 2$
   $y = 3(x^2 + \frac{5}{3}x + \frac{25}{36}) - 2 - 3\left(\frac{25}{36}\right)$
   $y = 3(x + \frac{5}{6})^2 - \frac{49}{12}$ Vertex $(-\frac{5}{6}, -4\frac{1}{12})$
   x-intercept (y=0): $0 = 3x^2 + 5x - 2$
   $0 = (3x - 1)(x + 2)$
   $x = \frac{1}{3}$ or -2
   $(-2,0)$, $(\frac{1}{3},0)$
   y-intercept (x=0): $y = -2$ $(0,-2)$

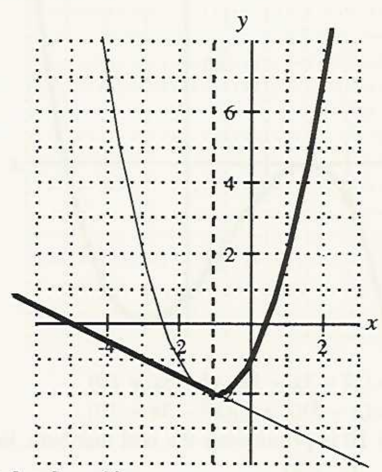


5. Let x be one of the sides as shown in the diagram. Then there is $50 - 2x$ feet left for the other side. The area, A, is $x(50 - 2x)$. We maximize area by finding the vertex of the parabola $A = 50x - 2x^2$.
   $A = -2x^2 + 50x$
   $= -2(x^2 - 25x)$
   $= -2\left(x^2 - 25x + \left(\frac{25}{2}\right)^2\right) + 2\left(\frac{25}{2}\right)^2$
   $= -2(x - \frac{25}{2})^2 + \frac{625}{2}$



Thus the vertex is at the point $(x, A) = (12.5, 312.5)$, which means the dimensions should be 12.5' by 25', and the area will be 312.5 square feet.

7. $g(x) = \begin{cases} -\frac{1}{2}x - \frac{5}{2}, x < -1 \\ x^2 + 2x - 1, x \geq -1 \end{cases}$
   Graph the straight line $y = -\frac{1}{2}x - \frac{5}{2}$ and the parabola $y = x^2 + 2x - 1$ in the same graph.
   $y = -\frac{1}{2}x - \frac{5}{2}$
   x-intercept: $0 = -\frac{1}{2}x - \frac{5}{2}$
   $x = -5$
   y-intercept: $y = -\frac{5}{2}$
   $y = x^2 + 2x - 1$
   $y = (x + 1)^2 - 2$ Vertex $(-1,2)$
   x-intercept: $0 = (x + 1)^2 - 2$
   $2 = (x + 1)^2$
   $x = -1 \pm \sqrt{2} \approx -2.4, -0.4$
   y-intercept: $y = -1$



9. $4x^3 - 4x^2 + 2x - 12$
   Numerator: 1, 2, 3, 4, 6, 12
   Denominator: 1, 2, 4
   $\pm (1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2}, \frac{1}{4}, \frac{3}{4})$

11. $f(x) = 4x^3 - 4x^2 - x + 1$
   (a) $f(x)$: 2 sign changes, so 0 or 2 positive real zeros.
   $f(-x) = -4x^3 - 4x^2 + x + 1$
   1 sign change, so there is exactly one negative real zero.
   (b) Possible rational zeros: $\pm (1, \frac{1}{2}, \frac{1}{4})$
   (c,d) Synthetic division may be used, but the expression for f may also be factored using grouping.
   $f(x) = 4x^3 - 4x^2 - x + 1$
   $= 4x^2(x - 1) - 1(x - 1)$
   $= (x - 1)(4x^2 - 1)$
   $= (x - 1)(2x - 1)(2x + 1)$
   Thus $f(x) = (x - 1)(2x - 1)(2x + 1)$, and all the real zeros are $1, \pm \frac{1}{2}$.

13. $h(x) = 3x^5 - 5x^4 - 23x^3 + 53x^2 - 16x - 12$
(a) $f(x)$: 3 sign changes, so 1 or 3 positive real zeros.
$f(-x) = -3x^5 - 5x^4 + 23x^3 + 53x^2 + 16x - 12$
2 sign changes, so there are 0 or 2 negative real zeros.
(b) $\pm(1, 2, 3, 4, 6, 12, \frac{1}{3}, \frac{2}{3}, \frac{4}{3})$
(c,d) Synthetic division shows that 1, 2, –3 are zeros, leaving a quadratic expression.

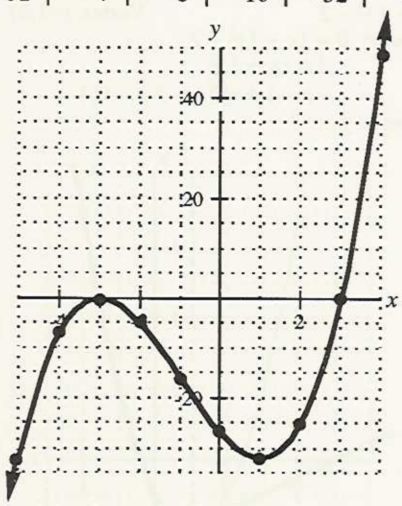
$$f(x) = (x - 1)(x - 2)(x + 3)(3x^2 - 5x - 2)$$
$$= (x - 1)(x - 2)(x + 3)(3x + 1)(x - 2)$$

Thus $f(x) = (x - 1)(x - 2)^2(x + 3)(3x + 1)$, and the real zeros are $1, 2, -3, -\frac{1}{3}$.

The zero 2 has multiplicity 2.

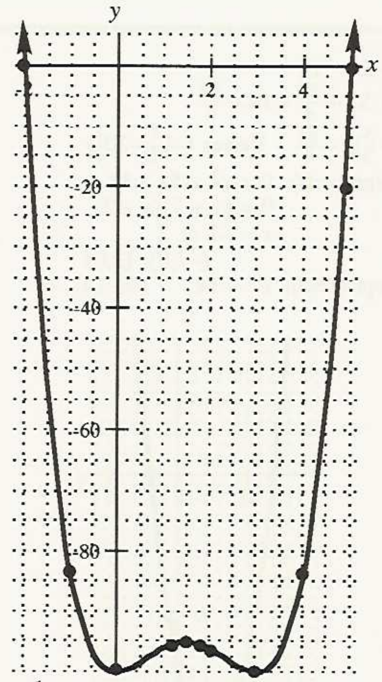
15. $g(x) = x^3 + 3x^2 - 9x - 27$
The expression for $g(x)$ can be factored (by grouping), or synthetic division used, to show that
$g(x) = (x + 3)^2(x - 3)$.
x-intercepts: 3, –3. At –3 there is even multiplicity, so the graph does not cross the x-axis there.
y-intercept: $f(0) = -27$
Additional Points:

x	–5	–4	–2	–1	1	2	4
y	–32	–7	–5	–16	–32	–25	49



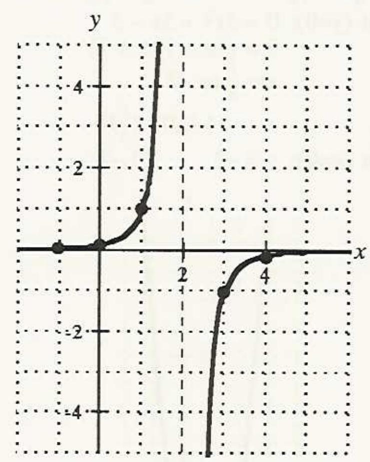
17. $h(x) = (x^2 - 3x - 10)(x^2 - 3x + 10)$
$= (x - 5)(x + 2)(x^2 - 3x + 10)$
$x^2 - 3x + 10$ is prime over the real numbers, but its complex zeros are $1\frac{1}{2} \pm \frac{\sqrt{31}}{2}i$, so we should plot extra points near the real component of these zeros, $1\frac{1}{2}$.
x-intercepts at –2, 5.
y-intercept = $h(0) = -100$.
Additional Points:

x	–3	–1	1	1.25	1.5	1.75	2	3	4	6
y	224	–84	–96	–95.2	–94.9	–95.2	–96	–100	–84	224



19. $f(x) = \dfrac{-1}{(x - 2)^3}$
This is like $y = \frac{1}{x^3}$, which is like $y = \frac{1}{x}$, except flipped over and shifted right two units.
y-intercept: $f(0) = \frac{1}{8}$.
Additional Points:

x	–1	1	3	4
y	$\frac{1}{27}$	1	–1	$-\frac{1}{8}$



21. $f(x) = \dfrac{-x}{x^2 - 4}$

$= \dfrac{-x}{(x-2)(x+2)}$

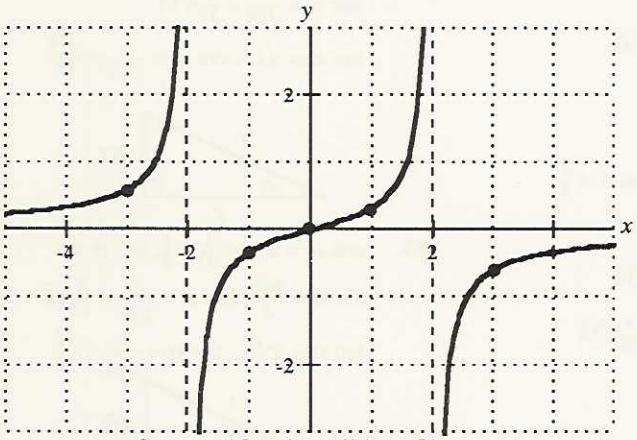
Vertical asymptotes at ± 2.

x-intercept: $0 = \dfrac{-x}{x^2 - 4}$, so $x = 0$ (the origin is an intercept).

y-intercept: $f(0) = 0$ (the origin).

Additional Points:

x	−3	−1	1	3
y	0.6	−0.3	−0.6	0.3



23. $f(x) = \dfrac{x^2 - x - 12}{x - 5} = \dfrac{(x-4)(x+3)}{x-5}$

$\dfrac{x^2 - x - 12}{x - 5} = x + 4 + \dfrac{8}{x-5}$, so the line $x + 4$ is a slant asymptote.

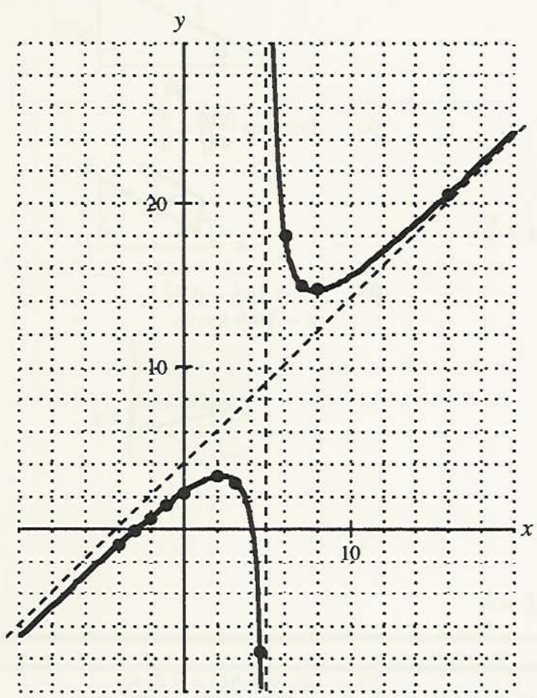
Vertical asymptote at $x = 5$.

x-intercepts are at −3 and 4.

y-intercept at $f(0) = 2\frac{2}{5}$.

Additional Points:

x	−4	−2	−1	2	3	4.5	6	7	8	16
y	−0.9	0.9	1.7	3.3	3	−7.5	18	15	14.7	20.7



25. $f(x) = x^4 - 2$; $g(x) = 2\sqrt{x+1}$

$(f + g)(x) = f(x) + g(x) = x^4 - 2 + 2\sqrt{x+1}$

$(f - g)(x) = f(x) - g(x) = x^4 - 2 - 2\sqrt{x+1}$

$(f \cdot g)(x) = f(x) \cdot g(x) = 2(x^4 - 2)\sqrt{x+1}$

$(f / g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^4 - 2}{2\sqrt{x+1}}$

$(f \circ g)(x) = f(g(x)) = [g(x)]^4 - 2 = [2\sqrt{x+1}]^4 - 2 = 2^4[(x+1)^{1/2}]^4 - 2 = 16(x+1)^2 - 2 = 16x^2 + 32x + 14$

$(g \circ f)(x) = g(f(x)) = 2\backslash r(f(x) + 1) = 2\backslash r((x^4 - 2) + 1) = 2\sqrt{x^4 - 1}$

27. $g(x) = 5x + 4$

$y = 5x + 4$

$x = 5y + 4$

$x - 4 = 5y$

$\dfrac{x-4}{5} = f$

$g^{-1}(x) = \dfrac{x-4}{5}$

29. $g(x) = x^2 - 4,\ x \geq 0$

$y = x^2 - 4,\ x \geq 0$

$x = y^2 - 4,\ y \geq 0$

$x + 4 = y^2,\ y \geq 0$

$\pm\sqrt{x+4} = y$ Choose the "+" value so $y \geq 0$.

$g^{-1}(x) = \sqrt{x+4}$

31. We have two ordered (voltage, temperature) pairs: (60 mv, 50°) and (80 mv, 100°). To find the linear function we find the equation of the straight line which contains these points, and solve the equation for y.

$m = \dfrac{100 - 50}{80 - 60} = \dfrac{5}{2}$;

$y - 50 = \frac{5}{2}(x - 60)$

$y = \frac{5}{2}x - 150 + 50$

$f(x) = 2.5x - 100.$

To predict the temperature with an output of 65 mv, we compute f(65):

$f(65) = 2.5(65) - 100 = 62.5°$ F.

33. $\dfrac{4x^2 + 4x + 10}{(x-1)(x^2 + x + 4)} = \dfrac{A}{x-1} + \dfrac{Bx + C}{x^2 + x + 4}$

Multiply each term by the LCD, $(x - 1)(x^2 + x + 4)$:

$4x^2 + 4x + 10 = A(x^2 + x + 4) + (Bx + C)(x - 1)$

Let $x = 1$: $18 = A(6)$

$A = 3.$

Let $x = 0, A = 3$: $10 = 3(4) + C(-1)$

$C = 2.$

Let $x = 2, A = 3, C = 2$:

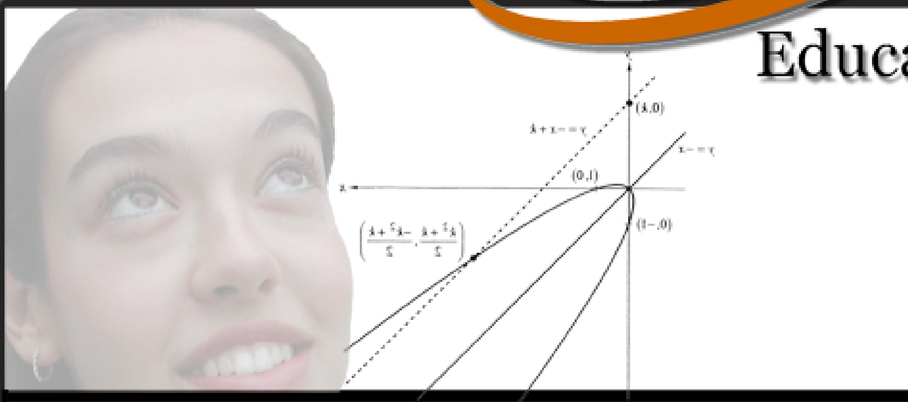
$34 = 3(10) + (2B + 2)(1)$

$34 = 30 + 2B + 2$

$B = 1.$

Thus $\dfrac{4x^2 + 4x + 10}{(x-1)(x^2 + x + 4)}$

$= \dfrac{3}{x-1} + \dfrac{x+2}{x^2 + x + 4}.$



Chapter 5

Exercise 5–1

1. $13° 25' = \left(13 + \frac{25}{60}\right)° \approx 13.417°$

5. $25° 33' 19'' = \left(25 + \frac{33}{60} + \frac{19}{3600}\right)° \approx$ $25.555°$

9. $33° 5' 55'' = \left(33 + \frac{5}{60} + \frac{55}{3600}\right)° \approx$ $33.099°$

13. $(180 - 43.2 - 88.6)° = 48.2°$

17. $(180 - 43.45 - 30.15)° = 106.40°$

21. $c^2 = a^2 + b^2$
$10^2 = a^2 + 8^2$
$100 = a^2 + 64$
$36 = a^2$
$a = \sqrt{36} = 6$

25. $c^2 = a^2 + b^2$
$c^2 = (\sqrt{5})^2 + 3^2$
$c^2 = 5 + 9$
$c^2 = 14$
$c = \sqrt{14} \approx 3.7$

29. $c^2 = a^2 + b^2$
$c^2 = (\sqrt{7})^2 + 3^2$
$c^2 = 7 + 9$
$c^2 = 16$
$c = 4$

33. $c^2 = a^2 + b^2$
$c^2 = (3\sqrt{2})^2 + (4\sqrt{5})^2$
$c^2 = 9{\cdot}2 + 16{\cdot}5$
$c^2 = 98$
$c = \sqrt{98}$
$c = 7\sqrt{2} \approx 9.9$

37. $c^2 = a^2 + b^2$
$c^2 = 1^2 + 1^2$
$c^2 = 2$
$c = \sqrt{2} \approx 1.4$

41. $213^2 = 193^2 + w^2$
$8120 = w^2$
90.1 feet $\approx w$

45. $Z^2 = R^2 + X_L^2$
$4340^2 = R^2 + 2150^2$
$14213100 = R^2$
3770 ohms $\approx R$

49. $125^2 = 30^2 + h^2$, so
$h \approx 121$ feet. The reach of the ladder decreases by about 1 foot, not 5 feet.

53. $\tan \beta = \sqrt{2}$
$\cot \beta = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

57. $\cos \theta = \frac{\sqrt{6}}{8}$ $\quad \sec \theta = \frac{8}{\sqrt{6}}$
$= \frac{8}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{8}{6}\sqrt{6} = \frac{4}{3}\sqrt{6}$

61. $c^2 = 4^2 + (\sqrt{10})^2$
$c = \sqrt{26}$
$\sin B = \frac{b}{c} = \frac{\sqrt{10}}{\sqrt{26}} = \sqrt{\frac{5}{13}} = \frac{\sqrt{5}}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{65}}{13}$
$\csc B = \sqrt{\frac{13}{5}} = \frac{\sqrt{13}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{65}}{5}$
$\cos B = \frac{a}{c} = \frac{4}{\sqrt{26}} = \frac{4}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{4}{26}\sqrt{26}$
$= \frac{2}{13}\sqrt{26}$
$\sec B = \frac{\sqrt{26}}{4}$ $\quad \tan B = \frac{b}{a} = \frac{\sqrt{10}}{4}$
$\cot B = \frac{4}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$
$= \frac{4}{10}\sqrt{10} = \frac{2}{5}\sqrt{10}$

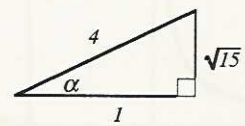
65. $4^2 + b^2 = 7^2$
$b^2 = 33$
$b = \sqrt{33}$
$\sin A = \frac{a}{c} = \frac{4}{7}$ $\quad \csc A = \frac{7}{4}$
$\cos A = \frac{b}{c} = \frac{\sqrt{33}}{7}$
$\sec A = \frac{7}{\sqrt{33}} = \frac{7}{33}\sqrt{33}$
$\tan A = \frac{a}{b} = \frac{4}{\sqrt{33}} = \frac{4}{33}\sqrt{33}$
$\cot A = \frac{\sqrt{33}}{4}$

69. $10^2 + b^2 = 15^2$
$b^2 = 125$
$b = \sqrt{5 \cdot 25} = 5\sqrt{5}$
$\sin A = \frac{a}{c} = \frac{10}{15} = \frac{2}{3}$ $\quad \csc A = \frac{3}{2}$
$\cos A = \frac{b}{c} = \frac{5\sqrt{5}}{15} = \frac{\sqrt{5}}{3}$
$\sec A = \frac{3}{\sqrt{5}} = \frac{3}{5}\sqrt{5}$
$\tan A = \frac{a}{b} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{5}\sqrt{5}$
$\cot A = \frac{\sqrt{5}}{2}$

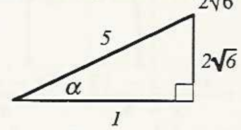
73. $a^2 + 5^2 = 8^2$
$a^2 = 39$
$a = \sqrt{39}$
$\sin A = \frac{a}{c} = \frac{\sqrt{39}}{8}$
$\csc A = \frac{8}{\sqrt{39}} = \frac{8}{39}\sqrt{39}$
$\cos A = \frac{b}{c} = \frac{5}{8}$ $\quad \sec A = \frac{8}{5}$
$\tan A = \frac{a}{b} = \frac{\sqrt{39}}{5}$
$\cot A = \frac{5}{\sqrt{39}} = \frac{5}{39}\sqrt{39}$

77. $x^2 + b^2 = z^2$
$b^2 = z^2 - x^2$
$b = \sqrt{z^2 - x^2}$
$\sin B = \frac{b}{c} = \frac{\sqrt{z^2 - x^2}}{z}$
$\csc B = \frac{z}{\sqrt{z^2 - x^2}}$
$\cos B = \frac{a}{c} = \frac{x}{z}$ $\quad \sec B = \frac{z}{x}$
$\tan B = \frac{b}{a} = \frac{\sqrt{z^2 - x^2}}{x}$
$\cot B = \frac{x}{\sqrt{z^2 - x^2}}$

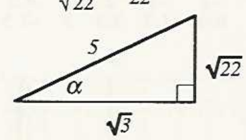
81. $\cos \alpha = \frac{1}{4}$ $\quad \sec \alpha = 4$
$\sin \alpha = \frac{\sqrt{15}}{4}$
$\csc \alpha = \frac{4}{\sqrt{15}} = \frac{4}{15}\sqrt{15}$
$\tan \alpha = \sqrt{15}$; $\cot \alpha = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$



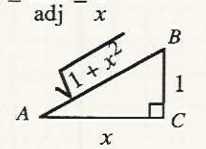
85. $\cos \alpha = 0.2 = \frac{2}{10} = \frac{1}{5}$; $\sec \alpha = 5$
$\sin \alpha = \frac{2\sqrt{6}}{5}$; $\csc \alpha = \frac{5}{2\sqrt{6}} = \frac{5}{12}\sqrt{6}$
$\tan \alpha = 2\sqrt{6}$; $\cot \alpha = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$



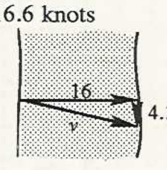
89. $\cos \alpha = \frac{\sqrt{3}}{5}$; $\sec \alpha = \frac{5}{\sqrt{3}} = \frac{5}{3}\sqrt{3}$
$\sin \alpha = \frac{\sqrt{22}}{5}$; $\csc \alpha = \frac{5}{\sqrt{22}} = \frac{5}{22}\sqrt{22}$
$\tan \alpha = \frac{\sqrt{22}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{66}}{3}$
$\cot \alpha = \frac{\sqrt{3}}{\sqrt{22}} = \frac{\sqrt{66}}{22}$



93. $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{1}{x}$



97. $v^2 = 16^2 + 4.3^2$
$v \approx 16.6$ knots



Exercise 5–2

1. $\sin 31.28° \approx 0.5192$

5. $\cot 28.87° \approx 1.8137$
28.87 [tan] [1/x]
[TI-81] [(] [TAN] 28.87 [)]
[x^{-1}] [ENTER]

9. $\sec 66.47° \approx 2.5048$

13. $\sin 35° 56' \approx 0.5868$
35 [+] 56 [÷] 60 [=] [sin]
[TI-81] [SIN] [(] 35 [+] 56 [÷] 60
[)] [ENTER]

17. $\tan 40°\,41' \approx 0.8596$

21. $\cot 13°\,3' \approx 4.3143$

13 $\boxed{+}$ 3 $\boxed{\div}$ 60 $\boxed{=}$ $\boxed{\tan}$ $\boxed{1/x}$

$\boxed{\text{TI-81}}$ $\boxed{(}$ $\boxed{\text{TAN}}$ $\boxed{(}$ 13 $\boxed{+}$ 3 $\boxed{\div}$

60 $\boxed{)}$ $\boxed{)}$ $\boxed{x^{-1}}$

25. $R = \dfrac{425}{2\sin 13.2°} \approx 930.6$ sq. ft.

29. $P = 120 \cdot 2.3 \cos 45° \approx 195.2$ watts

33. $36.8 = 2{\cdot}24 \cos\theta$

$\dfrac{36.8}{48} = \cos\theta;\ \theta \approx 39.9°$:

36.8 $\boxed{\div}$ 48 $\boxed{=}$ $\boxed{\text{shift}}$ $\boxed{\cos}$

$\boxed{\text{TI-81}}$ $\boxed{\text{2nd}}$ $\boxed{\text{COS}}$ $\boxed{(}$ 36.8 $\boxed{\div}$

48 $\boxed{)}$ $\boxed{\text{ENTER}}$

37. $\tan\theta = 1.8807 \quad \theta \approx 62.0°$

41. $\sin\theta = \dfrac{35.9}{68.3} \quad \theta \approx 31.7°$

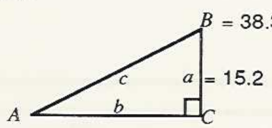
45. $\sec\theta = 4.8097 \quad \theta \approx 78.0°$

49. $a = 15.2,\ B = 38.3°$

$A = 90° - 38.3° = 51.7°$

$\cos 38.3° = \dfrac{15.2}{c};\ c = \dfrac{15.2}{\cos 38.3°} \approx 19.4$

$\tan 38.3° = \dfrac{b}{15.2};\ b = 15.2 \tan 38.3° \approx 12.0$

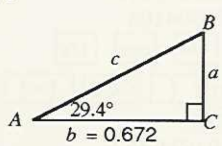


53. $b = 0.672,\ A = 29.4°$

$B = 90° - 29.4° = 60.6°$

$\tan 29.4° = \dfrac{a}{0.672};\ a = 0.672 \tan 29.4° \approx 0.379$

$\cos 29.4° = \dfrac{0.672}{c};\ c = \dfrac{0.672}{\cos 29.4°} \approx 0.771$

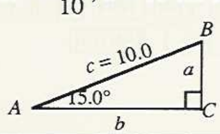


57. $c = 10.0,\ A = 15.0°$

$B = 90° - 15.0° = 75.0°$

$\cos 15° = \dfrac{b}{10};\ b = 10\cos 15° \approx 9.7$

$\sin 15° = \dfrac{a}{10};\ a = 10 \sin 15° \approx 2.6$

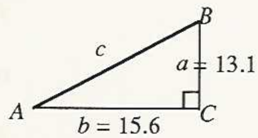


61. $a = 13.1 \quad b = 15.6$

$c^2 = \sqrt{13.1^2 + 15.6^2};\ c \approx 20.4$

$\tan A = \dfrac{13.1}{15.6};\ A \approx 40.0°$

$\tan B = \dfrac{15.6}{13.1};\ B \approx 50.0°$



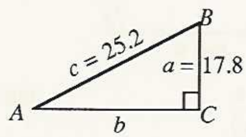
65. $a = 17.8 \quad c = 25.2$

$17.8^2 + b^2 = 25.2^2$

$b = \sqrt{25.2^2 - 17.8^2} \approx 17.8$

$\sin A = \dfrac{17.8}{25.2};\ A \approx 44.9°$

$\cos B = \dfrac{17.8}{25.2};\ B \approx 45.1°$

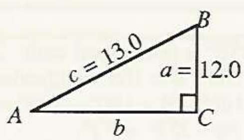


69. $a = 12.0 \quad c = 13.0$

$b = \sqrt{13^2 - 12^2} \approx 5.0$

$\sin A = \dfrac{12}{13};\ A \approx 67.4°$

$\cos B = \dfrac{12}{13};\ B \approx 22.6°$



73. $Z^2 = R^2 + X_L^2$

$10.35^2 = R^2 + 4.24^2$

$R = \sqrt{10.35^2 - 4.24^2} \approx 9.44$ ohms

$\sin\theta = \dfrac{4.24}{10.35};\ \theta \approx 24.2°$

77. $\sin\dfrac{\theta}{2} = \dfrac{\frac{b}{2}}{a};\ a = \dfrac{\frac{b}{2}}{\sin\frac{\theta}{2}};$

$\theta = \dfrac{360}{7}°$ so $\dfrac{\theta}{2} = \dfrac{1}{2} \cdot \dfrac{360}{7}°$

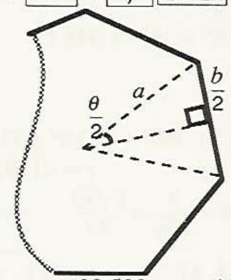
$= \dfrac{180}{7}° \cdot \dfrac{b}{2} = \dfrac{8\frac{1}{4}}{2} = 4\frac{1}{8} = 4.125.$

Thus, $a = \dfrac{\frac{b}{2}}{\sin\frac{\theta}{2}} = \dfrac{4.125}{\sin\left(\frac{180}{7}°\right)} \approx 9.51$ inches.

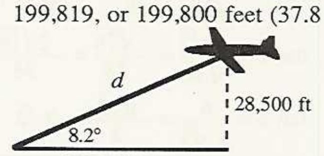
4.125 $\boxed{\div}$ $\boxed{(}$ 180 $\boxed{\div}$ 7 $\boxed{)}$ $\boxed{\sin}$ $\boxed{=}$

$\boxed{\text{TI-81}}$ 4.125 $\boxed{\div}$ $\boxed{\text{SIN}}$ $\boxed{(}$

180 $\boxed{\div}$ 7 $\boxed{)}$ $\boxed{\text{ENTER}}$



81. $\sin 8.2° = \dfrac{28{,}500}{d};\ d = \dfrac{28{,}500}{\sin 8.2°} \approx$ 199,819, or 199,800 feet (37.8 miles).



85. $a = \sqrt{122^2 + 83^2} \approx 147.5567687$ cm

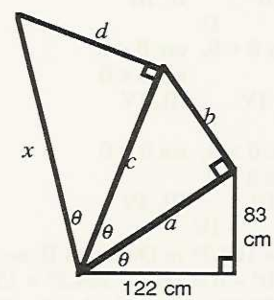
$\tan\theta = \dfrac{83}{122};\ \theta \approx 34.22854584°$

$\cos\theta = \dfrac{a}{c};\ c = \dfrac{a}{\cos\theta} =$

$\dfrac{147.5567687}{\cos 34.22854584°} \approx 178.4672131$ cm

$\cos\theta = \dfrac{c}{x};\ x = \dfrac{c}{\cos\theta} =$

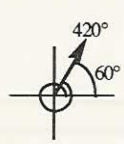
$\dfrac{178.4672131}{\cos 34.22854584°} \approx 215.8528302$ cm. Thus $x \approx 215.9$ cm.



Exercise 5–3

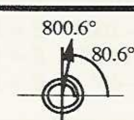
1. 420°

$420° - 360° = 60°$



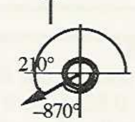
5. 800.6°

$800.6° - 2 \cdot 360° = 80.6°$



9. −870°

$-870° + 3 \cdot 360° = 210°$



13. $-530.3° + 2 \cdot 360° = 189.7°$

17. $-11.9°$
$-11.9° + 360° = 348.1°$

21. $-6.1° + 360° = 353.9°$ ATDC

25. $(-5, 8)$ $\quad r = \sqrt{(-5)^2 + 8^2} = \sqrt{89}$

$\sin\theta = \dfrac{8}{\sqrt{89}} = \dfrac{8\sqrt{89}}{89} \qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{89}}{8}$

$\cos\theta = \dfrac{-5}{\sqrt{89}} = \dfrac{-5\sqrt{89}}{89} \qquad \sec\theta = \dfrac{1}{\cos\theta} = -\dfrac{\sqrt{89}}{5}$

$\tan\theta = \dfrac{8}{-5} = -1\tfrac{3}{5} \qquad \cot\theta = \dfrac{1}{\tan\theta} = -\dfrac{5}{8}$

29. $(-1, 4)$ $\quad r = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

$\sin\theta = \dfrac{4}{\sqrt{17}} = \dfrac{4\sqrt{17}}{17} \qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{17}}{4}$

$\cos\theta = \dfrac{-1}{\sqrt{17}} = -\dfrac{\sqrt{17}}{17} \qquad \sec\theta = \dfrac{1}{\cos\theta} = -\sqrt{17}$

$\tan\theta = \dfrac{4}{-1} = -4 \qquad \cot\theta = \dfrac{1}{\tan\theta} = -\dfrac{1}{4}$

33. $(-\sqrt{2}, 6)$ $\quad r = \sqrt{(-\sqrt{2})^2 + 6^2} = \sqrt{38}$

$\sin\theta = \dfrac{6}{\sqrt{38}} = \dfrac{6\sqrt{38}}{38} = \dfrac{3\sqrt{38}}{19} \qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{38}}{6}$

$\cos\theta = \dfrac{-\sqrt{2}}{\sqrt{38}} = -\dfrac{\sqrt{76}}{38} = -\dfrac{2\sqrt{19}}{38} = -\dfrac{\sqrt{19}}{19} \qquad \sec\theta = \dfrac{1}{\cos\theta} = -\dfrac{\sqrt{38}}{\sqrt{2}} = -\dfrac{\sqrt{76}}{2} = -\dfrac{2\sqrt{19}}{2} = -\sqrt{19}$

$\tan\theta = \dfrac{6}{-\sqrt{2}} = -\dfrac{6\sqrt{2}}{2} = -3\sqrt{2} \qquad \cot\theta = \dfrac{1}{\tan\theta} = -\dfrac{\sqrt{2}}{6}$

37. $(\sqrt{6}, -\sqrt{10})$ $\quad r = \sqrt{(\sqrt{6})^2 + (-\sqrt{10})^2} = \sqrt{16} = 4$

$\sin\theta = -\dfrac{\sqrt{10}}{4} \qquad \csc\theta = -\dfrac{4}{\sqrt{10}} = -\dfrac{4\sqrt{10}}{10} = -\dfrac{2\sqrt{10}}{5}$

$\cos\theta = \dfrac{\sqrt{6}}{4} \qquad \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{4}{\sqrt{6}} = \dfrac{4\sqrt{6}}{6} = \dfrac{2\sqrt{6}}{3}$

$\tan\theta = -\dfrac{\sqrt{10}}{\sqrt{6}} = -\dfrac{\sqrt{60}}{6} = -\dfrac{2\sqrt{15}}{6} = -\dfrac{\sqrt{15}}{3}$

$\cot\theta = \dfrac{1}{\tan\theta} = -\dfrac{3}{\sqrt{15}} = -\dfrac{3\sqrt{15}}{15} = -\dfrac{\sqrt{15}}{5}$

41. $\cos\theta = \dfrac{x}{r} = \dfrac{1}{\frac{r}{x}} = \dfrac{1}{\sec\theta}$

Exercise 5-4

1. $\sin\theta > 0, \ \cos\theta < 0$
I, II $\quad$ II, III
II

5. $\tan\theta < 0, \ \csc\theta < 0$
$\sin\theta < 0$
II, IV $\quad$ III, IV
IV

9. $\sec\theta > 0, \ \sin\theta < 0$
$\cos\theta > 0$
I, IV $\quad$ III, IV
IV

13. $\theta = 164.2°$ in Quadrant II, so $\theta' = 180° - \theta = 180° - 164.2° = 15.8°$.

17. $-255.3°$ is coterminal with $-255.3° + 360° = 104.7°$. $\theta = 104.7°$ in Quadrant II, so $\theta' = 180° - \theta = 180° - 104.7° = 75.3°$. Thus θ' for $-255.3°$ is $75.3°$.

21. $-181.0°$ is coterminal with $-181.0° + 360° = 179.0°$. $\theta = 179.0°$ in Quadrant II, so $\theta' = 180° - \theta = 180° - 179.0° = 1.0°$.
Thus θ' for $-181.0°$ is $1.0°$.

25. $-252°$ is coterminal with $-252° + 360° = 108°$. $\theta = 108°$ in Quadrant II, so $\theta' = 180° - \theta = 180° - 108° = 72°$. Thus θ' for $-252°$ is $72°$.

29. $\theta' = 30°$; $\sin 30° = \frac{1}{2}$. In QIII, so $\sin 210° < 0$: $\sin 210° = -\frac{1}{2}$.

33. $\sin(-120°)$ $\qquad \theta' = 60°$; $\sin 60° = \frac{\sqrt{3}}{2}$. In QIII, so $\sin(-120°) < 0$:
$\sin(-120°) = -\frac{\sqrt{3}}{2}$

37. $\theta' = 60°$; $\cot 60° = \dfrac{1}{\tan 60°} = \dfrac{\sqrt{3}}{3}$. $300°$ is in Quadrant IV, where cotangent is negative, so
$\cot 300° = -\frac{\sqrt{3}}{3}$.

41. $\csc 90° = \dfrac{1}{\sin 90°} = 1$

45. $\theta' = 30°$. $\cos 30° = \frac{\sqrt{3}}{2}$; $150°$ in Quadrant II, so $\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$. $\sec 150°$
$= \dfrac{1}{\cos 150°} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$.

49. 0.6898537919

53. 1.026304108
13 [+/−] [cos] [1/x]
[TI-81] [( ] [COS] [(−)] 13 [ )] [x^{-1}] [ENTER]

57. − 1.036744916

61. $\theta = \sin^{-1} 0.25 \approx 14.5°$

65. $\theta = \tan^{-1}\left(-\frac{8}{5}\right) \approx -57.995° \approx -58.0°$
8 [÷] 5 [=] [±] [SHIFT] [tan]
[TI-81] [2nd] [TAN] [( ] [(−)] 8 [÷] 5 [ )] [ENTER]

69. $E = 156 \sin(\theta + 45°)$

(a) $\theta = 0°$	$E = 156 \sin(0° + 45°)$	$= 156 \sin 45°$	$\approx$	110.31
(b) $\theta = 45°$	$E = 156 \sin(45° + 45°)$	$= 156 \sin 90°$	$=$	156
(c) $\theta = 100°$	$E = 156 \sin(100° + 45°)$	$= 156 \sin 145°$	$\approx$	89.48
(d) $\theta = -200°$	$E = 156 \sin(-200° + 45°)$	$= 156 \sin(-155°)$	$\approx$	−65.93
(e) $\theta = 13.3°$	$E = 156 \sin(13.3° + 45°)$	$= 156 \sin 58.3°$	$\approx$	132.73
(f) $\theta = -45°$	$E = 156 \sin(-45° + 45°)$	$= 156 \sin 0°$	$=$	0

Exercise 5–5

1. $\sin\theta = 0.8251, \ \cos\theta > 0$
$\sin\theta' = 0.8251$, so $\theta' \approx 55.6°$
Since $\sin\theta > 0, \cos\theta > 0$, θ is in Quadrant I.
In quadrant I $\theta = \theta'$, so $\theta \approx 55.6°$.

5. $\sec\theta = -1.0642, \ \sin\theta < 0$
$\cos\theta = -\dfrac{1}{1.0642}$ so $\cos\theta' =$

$\frac{1}{1.0642}$; θ' ≈ 20.0°.
cos θ < 0, sin θ < 0, so θ is in Quadrant III.
θ = 180° + θ' ≈ 180° + 20.0° ≈ 200.0°.

9. $\sin\theta = \frac{3}{8}$, $\cos\theta > 0$

$\sin\theta' = \frac{3}{8}$, so θ' ≈ 22.0°.

sin θ > 0, cos θ > 0, so θ is in Quadrant I, and θ = θ' ≈ 22.0°.

13. $\cos\theta = -\frac{5}{7}$, $\tan\theta > 0$

$\cos\theta' = \frac{5}{7}$, so θ' ≈ 44.4°.

cos θ < 0, tan θ > 0 so θ is in Quadrant III.
θ = 180° + θ' ≈ 180° + 44.4° ≈ 224.4°.

17. (12, –5) ; $r = \sqrt{12^2 + (-5)^2} = 13$

$\sin\theta = \frac{y}{r} = -\frac{5}{13}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{13}{5}$

$\cos\theta = \frac{x}{r} = \frac{12}{13}$ $\quad \sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$

$\tan\theta = \frac{y}{x} = -\frac{5}{12}$

$\cot\theta = \frac{1}{\tan\theta} = -\frac{12}{5}$

$\sin\theta' = \frac{5}{13}$ so θ' ≈ 22.6°.

(12, –5) is in Quadrant IV, so θ terminates in Quadrant IV.
Thus θ = 360° – θ' ≈ 360° – 22.6° ≈ 337.4°.

21. (4, 6) ; $r = \sqrt{4^2 + 6^2} = 2\sqrt{13}$

$\sin\theta = \frac{y}{r} = \frac{6}{2\sqrt{13}} = \frac{6\sqrt{13}}{26} = \frac{3\sqrt{13}}{13}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{13}}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{2\sqrt{13}} = \frac{4\sqrt{13}}{26} = \frac{2\sqrt{13}}{13}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{13}}{2}$

$\tan\theta = \frac{y}{x} = \frac{6}{4} = \frac{3}{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{2}{3}$

$\sin\theta' = \frac{\sqrt{13}}{13}$ so θ' ≈ 56.3°.

(4, 6) is in Quadrant I, so θ terminates in Quadrant I. Thus θ = θ' ≈ 56.3°.

25. $\cos\theta = -\frac{1}{2}$, $\tan\theta > 0$

cos θ < 0, tan θ > 0 so θ in Quadrant III.

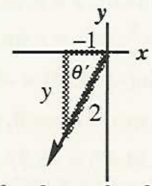
$y = -\sqrt{2^2 - (-1)^2} = -\sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{2} = -\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{-1} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

$\cos\theta' = \frac{1}{2}$, so θ' = 60°.

θ = 180° + θ' = 180° + 60° = 240°.



29. tan θ = 2, cos θ < 0

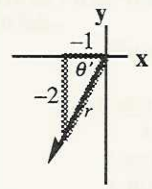
tan θ > 0, cos θ < 0 so θ in Quadrant III.

$r = +\sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-2}{r} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{r} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

tan θ' = 2, so θ' ≈ 63.4°.
θ = 180° + θ' ≈ 180° + 63.4° ≈ 243.4°.

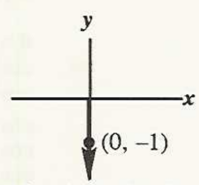


33. csc θ = –1 ; $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-1} =$ –1 ; θ is 270°; pick a point, say (0, –1) on the terminal side of the angle.

$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-1)^2} = 1$

$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$

$\tan\theta = \frac{y}{x} = \frac{-1}{0}$ (undefined)



37. sec θ = 4, csc θ > 0

$\cos\theta = \frac{1}{4}$, sin θ > 0

cos θ > 0, sin θ > 0 so θ in Quadrant I.

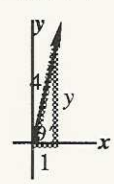
$y = +\sqrt{4^2 - 1^2} = \sqrt{15}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{4} = \frac{\sqrt{15}}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{1} = \sqrt{15}$

$\cos\theta = \frac{1}{4}$, so θ ≈ 75.5°.

θ = θ' ≈ 75.5°.



41. $\cos\theta = -\frac{5}{13}$, $\sin\theta > 0$

cos θ < 0, sin θ > 0, so θ in Quadrant II.

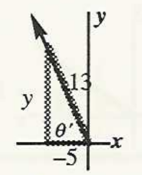
$y = +\sqrt{13^2 - (-5)^2} = 12$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{13} = \frac{12}{13}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{-5} = -\frac{12}{5}$.

$\cos\theta' = \frac{5}{13}$ so θ' ≈ 67.4°.

θ = 180° – θ' ≈ 180° – 67.4° ≈ 112.6°.



45. sec θ = 5, tan θ > 0

$\cos\theta = \frac{1}{5}$, tan θ > 0

cos θ > 0, tan θ > 0 so θ in Quadrant I.

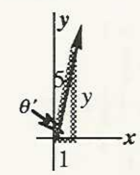
$y = +\sqrt{5^2 - 1^2} = 2\sqrt{6}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{5} = \frac{2\sqrt{6}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{1} = y = 2\sqrt{6}$.

$\cos\theta' = \frac{1}{5}$ so θ' ≈ 78.5°.

θ = θ' ≈ 78.5°.



49. cos θ = u and θ terminates in quadrant I.

$y = \sqrt{1^2 - u^2} = \sqrt{1 - u^2}$.

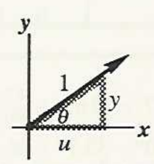
$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{u}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$
$= \sqrt{1 - u^2}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\sqrt{1 - u^2}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{u} = \frac{\sqrt{1 - u^2}}{u}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{u}{\sqrt{1 - u^2}}$.



53. sin θ = $u + 1$ and θ terminates in quadrant I.

$x = \sqrt{1^2 - (u + 1)^2}$
$= \sqrt{-u^2 - 2u}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{u + 1}$

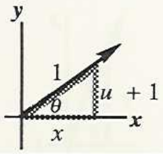
$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$
$= \sqrt{-u^2 - 2u}$; $\sec\theta = \frac{1}{\cos\theta}$

$$= \frac{1}{\sqrt{-u^2 - 2u}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{u+1}{x}$$

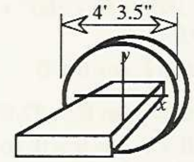
$$= \frac{u+1}{\sqrt{-u^2-2u}}$$

$$\cot\theta = \frac{\sqrt{-u^2-2u}}{u+1}.$$



57. $\theta = -134.4°$, $r = 8.25$.
$\sin\theta = \frac{y}{r}$, so $y = r\sin\theta$, so $y = 8.25\sin(-134.4°) \approx -5.89$ cm. $\cos\theta = \frac{x}{r}$, so $x = r\cos\theta$, so $x = 8.25\cos(-134.4°) \approx -5.77$ cm

61. 4' 3.5" = 4\f(3.5,12)" ≈ 4.292' ; $r \approx \frac{4.292}{2}$ ft ≈ 2.146 ft;
$\theta = 211.5°$. $\sin\theta = \frac{y}{r}$; $y = r\sin\theta$;
$y \approx 2.146\sin 211.5° \approx -1.121$ ft. 0.121 ft x 12"/ft ≈ 1.5", so $y \approx$ −1' 1.5"
$\cos\theta = \frac{x}{r}$; $x = r\cos\theta$; $x \approx 2.146\cos 211.5° \approx -1.830$ ft. 0.830 ft x 12"/ft ≈ 10.0", so $x \approx$ −1' 10.0"



Exercise 5–6

1. $\tan\theta\cot\theta$
$\tan\theta \cdot \frac{1}{\tan\theta}$
1

5. $\sec\theta(\cot\theta + \cos\theta - 1)$
$\sec\theta \cdot \cot\theta + \sec\theta \cdot \cos\theta - \sec\theta$
$\frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta}\cdot\cos\theta - \sec\theta$
$\frac{1}{\sin\theta} + 1 - \sec\theta$
$\csc\theta - \sec\theta + 1$

9. $1 - \cos^2\theta$
$(\sin^2\theta + \cos^2\theta) - \cos^2\theta$
$\sin^2\theta$

13. $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) + 2\sin^2\theta$
$\cos^2\theta - \cos\theta \cdot \sin\theta + \sin\theta\cos\theta - \sin^2\theta + 2\sin^2\theta$
$\cos^2\theta + \sin^2\theta$
1

17. $\frac{\sin x - \cos x}{\sin x}$
$\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}$
$1 - \cot x$

21. (a) $\theta = 16°\,50'$
$\sin^2\theta + \cos^2\theta = 1$
$\sin^2 16°50' + \cos^2 16°50' = 1$
$(0.28959)^2 + (0.95715)^2 = 1$
$1 = 1$
(b) $\theta = 50°$.
$\sin^2\theta + \cos^2\theta = 1$
$\sin^2 50° + \cos^2 50° = 1$
$(0.76604)^2 + (0.64279)^2 = 1$
$1 = 1$
The accuracy of these results depends on how much accuracy is used on a calculator.

25. $2\cos x = 1$
$\cos x = \frac{1}{2}$
$x = \cos^{-1}\frac{1}{2} = 60°$

29. $5\sin x = 1$
$\sin x = \frac{1}{5}$
$x = \sin^{-1}\frac{1}{5} \approx 11.5°$

33. $\frac{\sin x}{3} = \frac{2}{11}$
$\sin x = \frac{6}{11}$
$x = \sin^{-1}\frac{6}{11} \approx 33.1°$

37. $\tan 2x = \sqrt{3}$
$2x = \tan^{-1}\sqrt{3}$
$2x = 60°$
$x = 30°$

41. $2\cos 4x = -1$
$\cos 4x = -\frac{1}{2}$
$(4x)' = \cos^{-1}\frac{1}{2}$
$(4x)' = 60°$
Since $\cos 4x < 0$, $4x$ is in Quadrant II.
Thus $4x = 180° - (4x)'$
$= 180° - 60° = 120°$
$4x = 120°$
$x = 30°$

45. $4\sin^2\theta - 1 = 0$
$4\sin^2\theta = 1$
$\sin^2\theta = \frac{1}{4}$
$\sin\theta = \pm\sqrt{\frac{1}{4}}$
$\sin\theta = \pm\frac{1}{2}$

$\sin\theta = \frac{1}{2}$	$\sin\theta = -\frac{1}{2}$ (θ in Quadrant III)
$\theta = \sin^{-1}\frac{1}{2}$	$\sin\theta' = \frac{1}{2}$
$\theta = 30°$	$\theta' = 30°$
	$\theta = 180° + \theta' = 210°$

$\theta = 30°$ or $210°$

49. $\cos^2\theta - 1 = 0$
$\cos^2\theta = 1$
$\cos\theta = \pm\sqrt{1}$
$\cos\theta = \pm 1$
$\cos\theta = 1$ or $\cos\theta = -1$
$\theta = 0°$ or $\theta = 180°$
$\theta = 0°$ or $180°$

Chapter 5 Review

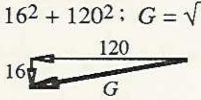
1. $165°\,47' = \left(165 + \frac{47}{60}\right)° \approx 165.783°$

3. $\theta = 180° - 35.7° - 66.1° = 78.2°$

5. $c^2 = a^2 + b^2$; $20^2 = 10^2 + b^2$; $20^2 - 10^2 = b^2$; $b^2 = 300$; $b = 10\sqrt{3} \approx 17.3$

7. $c^2 = a^2 + b^2$; $c^2 = 5^2 + 12^2 = 169$; $c = \sqrt{169} = 13$

9. $G^2 = 16^2 + 120^2$; $G = \sqrt{14656} \approx 121.0$ knots

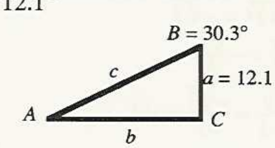


11. $\cot 58.7° = \frac{1}{\tan 58.7°} \approx 0.6080$

13. $\sec 4°38' = \frac{1}{\cos 4°38'} \approx 1.0033$

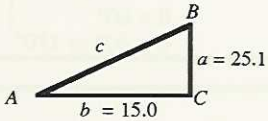
15. $\sin\theta = 0.215$ so $\theta = \sin^{-1}0.215 \approx 12.42°$

17. $a = 12.1$, $B = 30.3°$
$A = 90° - 30.3° = 59.7°$
$\cos 30.3° = \frac{12.1}{c}$; $c = \frac{12.1}{\cos 30.3°} \approx 14.0$
$\tan 30.3° = \frac{b}{12.1}$; $b = 12.1\tan 30.3° \approx 7.1$



19. $a = 25.1$, $b = 15.0$
$c^2 = 25.1^2 + 15^2$; $c = \sqrt{25.1^2 + 15^2} \approx 29.2$

$\tan A = \frac{25.1}{15.0}$; $A = \tan^{-1}\frac{25.1}{15.0} \approx 59.1°$
$\tan B = \frac{15.0}{25.1}$; $B = \tan^{-1}\frac{15.0}{25.1} \approx 30.9°$



21. $Z^2 = R^2 + X_L^2$; $60.0^2 = R^2 + 25.0^2$;
$R \approx 54.54$ ohms
$\sin\theta = \frac{X_L}{Z} = \frac{25.0}{60.0}$; $\theta \approx 24.6°$

23. $480° - 360° = 120°$

25. $1256° \div 360° \approx 3.5$
$1256° - 3(360°) = 176°$

27. $\cot\theta > 0, \cos\theta < 0$ so $\tan\theta > 0, \cos\theta < 0$
I or III II or III
III

29. $152.6°$ In quadrant II, so $\theta' = 180° - \theta = 180° - 152.6° = 27.4°$

31. $-13.22°$ $-13.22° + 360° = 346.78°$; in quadrant IV, so $\theta' = 360° - \theta = 360° - 346.78° = 13.22°$.

33. $-250°$ $-250° + 360° = 110°$, which is in quadrant II, so $\theta' = 180° - \theta = 180° - 110° = 70°$.

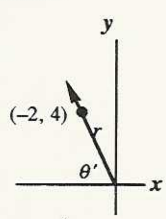
35. $\cos 48.5° \approx 0.6626$

37. $\csc 300° = \frac{1}{\sin 300°}$; $\theta' = 60°$; $\sin 60° = \frac{\sqrt{3}}{2}$. 300° is in quadrant IV, so $\sin 300° < 0$. Thus
$\sin 300° = -\frac{\sqrt{3}}{2}$, so $\csc 300° = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$.

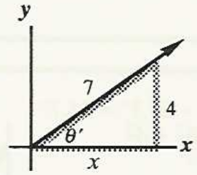
39. $\sin p = \frac{AB \sin b}{AP}$; $\sin p = \frac{420 \cdot \sin 20°}{410} \approx 0.35036$; $p \approx 20.509°$
$a = 180° - (b + p)$; $a = 180° - (20° + 20.509°) \approx 139.490°$
$BP = \frac{AP \sin a}{\sin b}$; $BP = \frac{410 \cdot \sin 139.490°}{\sin 20°} \approx 778.7$ ft.

41. $\sin\theta = -0.8133$, $\tan\theta > 0$
$\sin\theta' = 0.8133$, $\theta' = \sin^{-1}0.8133 \approx 54.4°$.
$\sin\theta < 0$, $\tan\theta > 0$, so θ is in quadrant III. Thus $\theta = 180° + \theta' \approx 180° + 54.4° \approx 234.4°$.

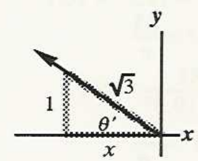
43. $(-2, 4)$ $r = \sqrt{(-2)^2 + 4^2} = 2\sqrt{5}$.
$\sin\theta = \frac{y}{r} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{5}}{2}$
$\cos\theta = \frac{x}{r} = \frac{-2}{2\sqrt{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$ $\sec\theta = \frac{1}{\cos\theta} = -\sqrt{5}$
$\tan\theta = \frac{y}{x} = \frac{4}{-2} = -2$ $\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{2}$
$\tan\theta' = -2$, so $\theta' = \tan^{-1} 2 \approx 63.4°$. $\theta = 180° - 63.4° \approx 116.6°$.



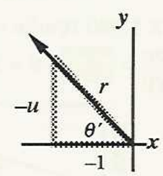
45. $\sin\theta = \frac{4}{7}$, $\cos\theta > 0$
$x = \sqrt{7^2 - 4^2} = \sqrt{33}$; $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{7} = \frac{\sqrt{33}}{7}$
$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{x}$
$= \frac{4}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$
$\theta' = \sin^{-1}\frac{4}{7} \approx 34.8°$.
$\theta = \theta' \approx 34.8°$



47. $\csc\theta = \sqrt{3}$, $\sec\theta < 0$
$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\sqrt{3}}$
$x = \sqrt{(\sqrt{3})^2 - 1^2} = \sqrt{2}$
$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$
$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\theta = \sin^{-1}\frac{1}{\sqrt{3}} \approx 35.3°$.
$\theta = 180° - \theta' \approx$
$180° - 35.3° \approx 144.7°$



49. $\tan\theta = u$ and θ terminates in quadrant II.
Since the bottom side must have length -1, the vertical side must be $-u$ so that $\tan\theta = \frac{-u}{-1} = u$.
$r = \sqrt{(-u)^2 + (-1)^2} = \sqrt{u^2 + 1}$
$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-u}{r} = \frac{-u}{\sqrt{u^2 + 1}}$
$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{r} = \frac{-1}{\sqrt{u^2 + 1}}$



51. $\sec\alpha(\cos\alpha - \cot\alpha)$
$\sec\alpha \cdot \cos\alpha - \sec\alpha \cdot \cot\alpha$
$\frac{1}{\cos\alpha} \cdot \cos\alpha - \frac{1}{\cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha}$
$1 - \frac{1}{\sin\alpha}$
$1 - \csc\alpha$

53. $\frac{\sin\beta - 1}{\cos\beta}$
$\frac{\sin\beta}{\cos\beta} - \frac{1}{\cos\beta}$
$\tan\beta - \sec\beta$

55. $\cot x\,(\sec x - \tan x + \frac{1}{\cot^2 x})$
$\cot x \cdot \sec x - \cot x \cdot \tan x + \cot x \cdot \frac{1}{\cot^2 x}$
$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} - \frac{1}{\tan x} \cdot \tan x + \frac{1}{\cot x}$
$\frac{1}{\sin x} - 1 + \tan x$
$\csc x - 1 + \tan x$

57. $2\sin x = 1$
$\sin x = \frac{1}{2}$
$x = \sin^{-1}\frac{1}{2} = 30°$

59. $3\tan x = 5$
$\tan x = \frac{5}{3}$
$x = \tan^{-1}\frac{5}{3} \approx 59.0°$

61. $\sec 3x = -2$
$\cos 3x = -\frac{1}{2}$
$(3x)' = \cos^{-1}\frac{1}{2} = 60°$

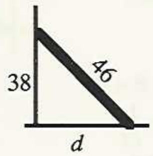
Since $\cos 3x < 0$, $3x$ terminates in Quadrant II. Thus $3x = 180° - (3x)'$:
$3x = 180° - 60° = 120°$
$x = 40°$

63. $2\cos^2\theta + \cos\theta - 1 = 0$
Substitution: $u = \cos\theta$:
$2u^2 + u - 1 = 0$
$(2u - 1)(u + 1) = 0$
$(2\cos\theta - 1)(\cos\theta + 1) = 0$

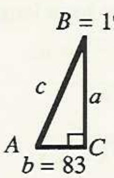
$2\cos\theta - 1 = 0$	$\cos\theta + 1 = 0$
$2\cos\theta = 1$	$\cos\theta = -1$
$\cos\theta = \frac{1}{2}$	$\theta = 180°$
$\theta = 60°$	
$\theta = 60°$ or $180°$	

Chapter 5 Test

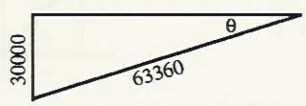
1. $\theta = 90° - 65.3° = 24.7°$
3. $46^2 = 38^2 + d^2$
$672 = d^2$
26 feet



5. $\sin 27.3°$ 0.4586
7. $\cot 38°10'$
$\frac{1}{\tan 38°10'}$ 1.2723
9. $y = 1.8\cos 6.2°\cos 21° - 1.8^2\cos 6.2°\sin 21° - 1.8^3\sin 6.2°$
$y \approx -0.11$
11. $A = 90° - 19.3° = 70.7°$
$\sin 19.3° = \frac{83}{c}$
$c = \frac{83}{\sin 19.3°} \approx 251.1$
$\tan 19.3° = \frac{83}{a}$
$a = \frac{83}{\tan 19.3°} \approx 237.0$



13. 12 miles x 5,280 ft/mile = 63,360 ft.
$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{30000}{63360}$; $\theta \approx 28.3°$

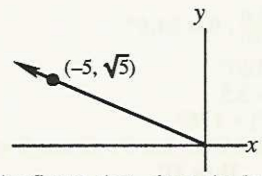


15.

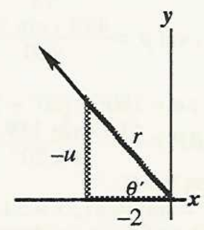
$\csc\theta > 0$	$\cos\theta < 0$
$\sin\theta > 0$	$\cos\theta < 0$
I or $\underline{\underline{\text{II}}}$	$\underline{\underline{\text{II}}}$ or III

II

17. $-192.1° + 360° = 167.9°$. $167.9°$ is in Quadrant II, so $\theta' = 180° - \theta = 180° - 167.9° = 12.1°$.
19. $\theta' = 45°$; $\cos 45° = \frac{\sqrt{2}}{2}$. $315°$ is in Quadrant IV, so $\cos 315° > 0$. Thus $\cos 315° = \frac{\sqrt{2}}{2}$. $\sec 315° = \frac{1}{\cos 315°} = \frac{2}{\sqrt{2}} = \sqrt{2}$.
21. $\sec 310° = \frac{1}{\cos 310°} \approx 1.5557$
23. $\sin\theta' = 0.4$, so $\theta' = \sin^{-1}0.4 \approx 23.6°$.
$\sin\theta < 0$ and $\tan\theta < 0$, so θ is in Quadrant IV. Thus $\theta = 360° - \theta' \approx 360° - 23.6° \approx 336.4°$.
25. $(-5, \sqrt{5})$
$r = \sqrt{(-5)^2 + (\sqrt{5})^2} = \sqrt{30}$.
$\sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{150}}{30} = \frac{5\sqrt{6}}{30} = \frac{\sqrt{6}}{6}$; $\csc\theta = \frac{1}{\sin\theta} = \frac{6}{\sqrt{6}} = \sqrt{6}$
$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{30}} = -\frac{5\sqrt{30}}{30} = -\frac{\sqrt{30}}{6}$; $\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{30}}{5}$
$\tan\theta = \frac{y}{x} = -\frac{\sqrt{5}}{5}$
$\cot\theta = \frac{1}{\tan\theta} = -\frac{5}{\sqrt{5}} = -\sqrt{5}$.
$\tan\theta' = |\tan\theta| = \frac{\sqrt{5}}{5}$ so $\theta' \approx 24.1°$
θ is in quadrant II so $\theta = 180° - \theta' \approx 180° - 24.1° \approx 155.9°$.



27. Observe in the figure that, since the length of the horizontal side must be -2, the vertical side must be $-u$, not u.
$r = \sqrt{(-u)^2 + (-2)^2} = \sqrt{u^2 + 4}$ $\sin\theta = \frac{-u}{r} = -\frac{u}{\sqrt{u^2 + 4}}$
$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{u^2 + 4}}{u}$
$\cos\theta = \frac{-2}{r} = -\frac{2}{\sqrt{u^2 + 4}}$
$\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{u^2 + 4}}{2}$
$\tan\theta = \frac{u}{2}$; $\cot\theta = \frac{1}{\tan\theta} = \frac{2}{u}$.



29. $\sec\theta(\cos\theta - \cos^3\theta)$
$\sec\theta \cdot \cos\theta - \sec\theta \cdot \cos^3\theta$
$\frac{1}{\cos\theta} \cdot \cos\theta - \frac{1}{\cos\theta} \cdot \cos^3\theta$
$1 - \cos^2\theta$
$\sin^2\theta + \cos^2\theta - \cos^2\theta$
$\sin^2\theta$
31. $2\sin x = \sqrt{2}$
$\sin x = \frac{\sqrt{2}}{2}$
$x = \sin^{-1}\frac{\sqrt{2}}{2} = 45°$
33. $\cos 2x = 0.62$
$2x = \cos^{-1}0.62$
$2x \approx 51.68°$
$x \approx 25.8°$.

Exercise 6-1

1. $x^2 + y^2 = 1$

5. $100°$ $\quad \frac{s}{\pi} = \frac{100°}{180°}$; $s = \frac{100° \cdot \pi}{180°} = \frac{5\pi}{9} \approx 1.75$

9. $270°$ $\quad \frac{s}{\pi} = \frac{270°}{180°}$; $s = \frac{270° \cdot \pi}{180°} = \frac{3\pi}{2} \approx 4.71$

13. $-305°$ $\quad \frac{s}{\pi} = \frac{-305°}{180°}$; $s = \frac{-305° \cdot \pi}{180°} = -\frac{61\pi}{36}$
≈ -5.32

In problems 14 through 30 we solve the definition of radians for θ°: $\frac{s}{\pi} = \frac{\theta°}{180°}$; $\boxed{\theta° = \frac{180°}{\pi} \cdot s}$.

17. $\frac{3\pi}{5}$ $\quad \theta° = \frac{180°}{\pi} \cdot \frac{3\pi}{5} = 108°$

21. $-\frac{17\pi}{6}$ $\quad \theta° = \frac{180°}{\pi} \cdot (-\frac{17\pi}{6}) = -510°$

25. $-\frac{12}{17}$ $\quad \theta° = \frac{180°}{\pi} \cdot (-\frac{12}{17}) = -\frac{2160}{17\pi} \approx -40.4°$

29. -5 $\quad \theta° = \frac{180°}{\pi} \cdot (-5) = -\frac{900}{\pi} \approx -286.5°$

$\boxed{\text{Make sure the calculator is in radian mode.}}$

33. $\tan 0.5 \approx 0.5463$

37. $\sin 2.3 \approx 0.7457$

41. $\csc 2.5 = \frac{1}{\sin 2.5} \approx 1.6709$

45. $\frac{11\pi}{6}$ is in quadrant IV, so $\cos \frac{11\pi}{6} > 0$, and $\theta' = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$. $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so $\cos \frac{11\pi}{6} = +\frac{\sqrt{3}}{2}$.

49. $\frac{5\pi}{4}$ is in quadrant III, so $\sin \frac{5\pi}{4} < 0$ and $\theta' = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$. $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$.

53. $r = 4.5$ mm, $L = 12$ mm;
$L = rs$; $12 = 4.5s$; $s \approx 2.7$ radians

57. Find L where $r = \frac{32.4}{2} = 16.2$ inches and
$\theta° = 85°$.
$85° = \frac{17}{36}\pi$
$L = rs$
$L = 16.2 \cdot \frac{17\pi}{36} \approx 24.0$ in

61. $V = 200 \sin (35t + 1)$
(a) $t = 0$: $V = 200 \sin (35(0) + 1)$
$= 200 \sin(1) \approx 168.3$ Volts
(b) $t = 0.1$: $V = 200 \sin (35(0.1) + 1)$
$= 200 \sin(4.5) \approx -195.5$ Volts
(c) $t = 0.8$: $V = 200 \sin (35(0.8) + 1)$
$= 200 \sin(29) \approx -132.7$ Volts
(d) $t = 1$: $V = 200 \sin (35(1) + 1)$
$= 200 \sin(36) \approx -198.4$ Volts

65. $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

			Using Calculator
(a) $x = 0.8$:	$\cos 0.8 \approx 1 - \frac{0.8^2}{2} + \frac{0.8^4}{24} - \frac{0.8^6}{720}$	≈ 0.6967025778	0.6967067093
(b) $x = 1$:	$\cos 1 \approx 1 - \frac{1^2}{2} + \frac{1^4}{24} - \frac{1^6}{720}$	≈ 0.5402777778	0.5403023059
(c) $x = 1.3$:	$\cos 1.3 \approx 1 - \frac{1.3^2}{2} + \frac{1.3^4}{24} - \frac{1.3^6}{720}$	≈ 0.2673002653	0.2674988286
(d) $10° = \frac{\pi}{18}$			
$x = \frac{\pi}{18}$:	$\cos \frac{\pi}{18} \approx 1 - \frac{(\frac{\pi}{18})^2}{2!} + \frac{(\frac{\pi}{18})^4}{4!} - \frac{(\frac{\pi}{18})^6}{6!}$	≈ 0.9848077530	0.9848077530

Note: To calculate part (d) most easily, compute $\frac{\pi}{18}$ and store it in the calculator's memory. Assuming [Min] is the key to enter a value into memory, and [MRec] is the key to recall the value in memory, then the following will calculate part d: [π] [÷] 18 [=] [Min] [−] [MRec] [x^y] 3 [÷] 6 [+] [MRec] [x^y] 5 [÷] 120 [−] [MRec] [x^y] 7 [÷] 5040 [=]

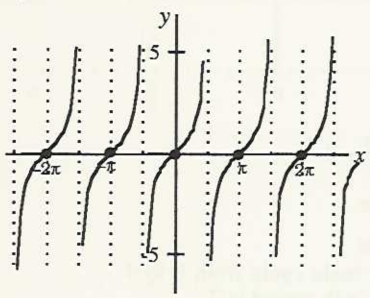
On the TI–81, store the equation as Y_1: [Y=] [CLEAR] [X|T] [−] [X|T] [MATH] 3 [÷] 18 [+] [X|T] [^] 5 [÷] 120 [−] [X|T] [^] 7 [÷] 5040 [2nd] [CLEAR]. To compute the value for say $x = -.1$, store this in X and calculate, as follows: 0.1 [STO▶] [X|T] [2nd] [VARS] 1 [ENTER]

69. $Ap = \frac{sr^2}{2} = \frac{2.4(5^2)}{2} = 30 \text{ in}^2$

73. $Ap = \frac{\theta°(\pi r^2)}{360°} = \frac{15° \cdot \pi \cdot 9^2}{360°} = \frac{27}{8}\pi \approx 10.60 \text{ mm}^2$

Exercise 6–2

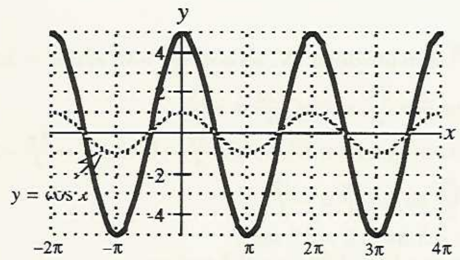
1. See figures 6.11, 6.14, 6.16.



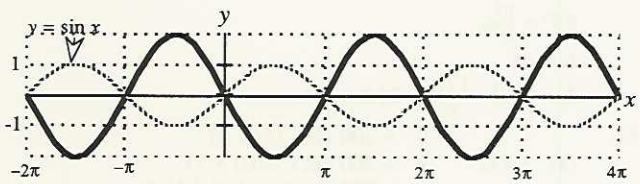
5. $k\pi$

9. $-\frac{\pi}{6}$ $\quad \sin(-\frac{\pi}{6}) = -\sin \frac{\pi}{6} = -\frac{1}{2}$
$\cos(-\frac{\pi}{6}) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$\tan(-\frac{\pi}{6}) = -\tan \frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

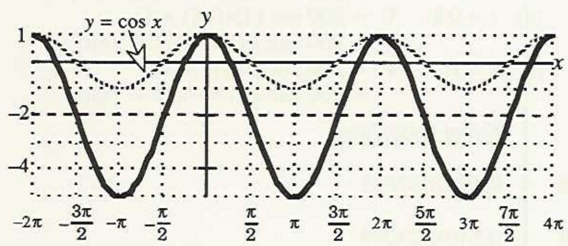
13. $y = 5 \cos x$ $\quad$ Amplitude is 5.



17. $y = -2 \sin x$ Amplitude is 2.
Graph is flipped about the x-axis relative to the graph of $y = \sin x$.



21. $y = 3 \cos x - 2$ Amplitude is 3.
Graph is lowered vertically 2 units.

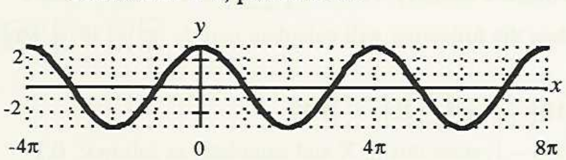


25. $y = 3 \cos \frac{x}{2}$ Amplitude is 3.

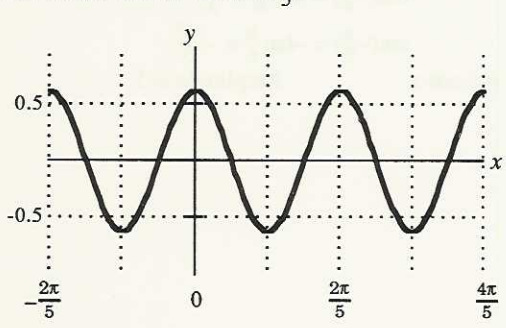
$0 \le \frac{x}{2} \le 2\pi$

$2(0) \le 2 \cdot \frac{x}{2} \le 2(2\pi)$

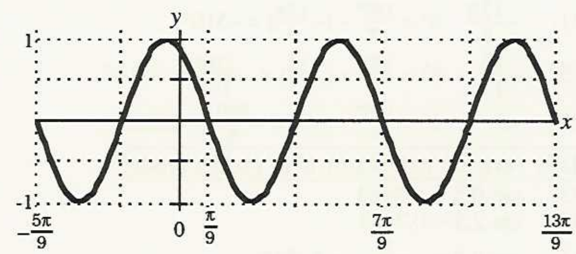
$0 \le x \le 4\pi$; one basic cosine cycle between 0 and 4π.
Phase shift is 0; period is 4π.



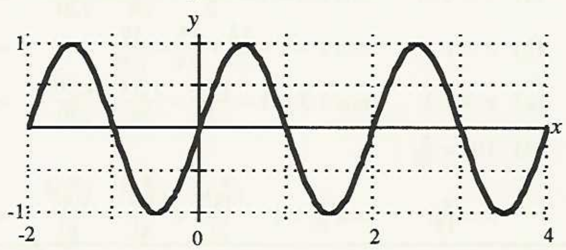
29. $y = \frac{5}{8} \cos 5x$ Amplitude is $\frac{5}{8}$.
$0 \le 5x \le 2\pi$
$0 \le x \le \frac{2\pi}{5}$ Divide each member by 5.
One basic cycle between 0 and $\frac{2\pi}{5}$.
Phase shift is 0; period is $\frac{2\pi}{5}$.
Mark the x-axis in terms of $\frac{\pi}{5}$.



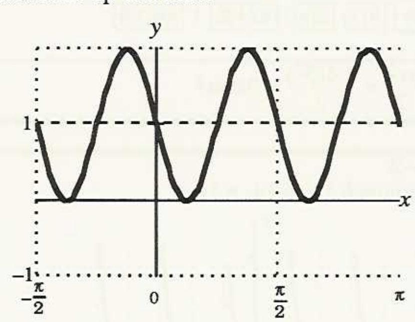
33. $y = -\sin\left(3x - \frac{\pi}{3}\right)$ Amplitude is 1.
Graph is reflected about the x-axis with respect to the graph of $y = \sin x$.
$0 \le 3x - \frac{\pi}{3} \le 2\pi$
$\frac{\pi}{3} \le 3x \le 2\pi + \frac{\pi}{3}$
$\frac{\pi}{3} \le 3x \le \frac{7\pi}{3}$
$\frac{1}{3} \cdot \frac{\pi}{3} \le \frac{1}{3} \cdot 3x \le \frac{1}{3} \cdot \frac{7\pi}{3}$
$\frac{\pi}{9} \le x \le \frac{7\pi}{9}$; one basic sine cycle between $\frac{\pi}{9}$ and $\frac{7\pi}{9}$.
Phase shift is $\frac{\pi}{9}$; period is $\frac{7\pi}{9} - \frac{\pi}{9} = \frac{2}{3}\pi$.



37. $y = \sin \pi x$ Amplitude is 1.
$0 \le \pi x \le 2\pi$
$0 \le x \le 2$ Divide each member by π.
Basic cycle from 0 to 2.
Phase shift is 0; period is $2 - 0 = 2$.

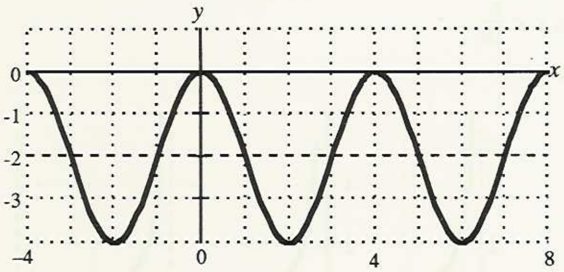


41. $y = -\sin 4x + 1$ Amplitude is 1.
The graph is flipped about the horizontal line $y = 1$.
$0 \le 4x \le 2\pi$
$0 \le x \le \frac{\pi}{2}$; one basic cycle from 0 to $\frac{\pi}{2}$.
Phase shift is 0; period is $\frac{\pi}{2}$.
Vertical shift up one unit.



45. $y = 2 \cos \frac{\pi x}{2} - 2$
$0 \le \frac{\pi x}{2} \le 2\pi$
$0 \le \pi x \le 4\pi$
$0 \le x \le 4$; basic cycle from 0 to 4.
Phase shift is 0; period is 2.

Vertical shift is two units downwards.
Mark the x-axis in units of 2.



49. $y = -\sin(-5x)$
$y = -[-\sin 5x]$
$y = \sin 5x$

53. $y = \cos(-2x) + 4$
$y = \cos 2x + 4$

65. amplitude = $|A| = 5$; $A = 5$. $D = 0$, since there is no vertical translation.
A basic sine cycle runs from -1 to 3.
To find B and C:

$-1 \le x \le 3$	Basic cycle. First convert the left member to 0.
$0 \le x + 1 \le 4$	Now convert the right member to 2π.
$0 \le \frac{x+1}{2} \le 2$	Divide each member by 2.
$0 \le \frac{x+1}{2}\pi \le 2\pi$	Multiply each member by π.
$0 \le \frac{\pi}{2}x + \frac{\pi}{2} \le 2\pi$	

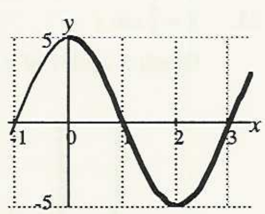
Thus $Bx + C = \frac{\pi}{2}x + \frac{\pi}{2}$, so $B = C = \frac{\pi}{2}$.

The equation is $y = 5\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

69. $A = 5, D = 0$
A cosine cycle begins at $x = 0$.
The period will be the same as that for the sine function, 4.
Thus, to find B and C:

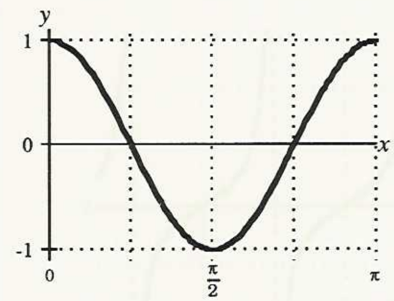
$0 \le x \le 4$	Basic cosine cycle.
$0 \le \frac{x}{2} \le 2$	Divide each member by 2.
$0 \le \frac{\pi x}{2} \le 2\pi$	Multiply each member by π

Thus $Bx + C = \frac{\pi x}{2}$, so $B = \frac{\pi}{2}$ and $C = 0$. The equation is $y = 5\cos\frac{\pi x}{2}$.

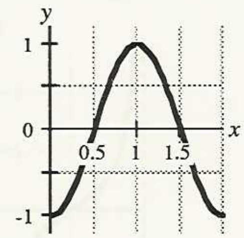


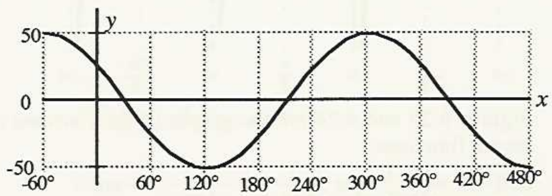
73. amplitude is 50. The graph is flipped about the horizontal (x) axis because the leading coefficient is negative.
$0° \le x - 120° \le 360°$
$120° \le x \le 480°$; one basic cosine cycle, flipped around the x-axis, from 120° to 480°.

57. $y = \cos(-2x)$
$y = \cos 2x$
$0 \le 2x \le 2\pi$
$0 \le x \le \pi$



61. $y = -\cos(-\pi x)$
$y = -\cos \pi x$
$0 \le \pi x \le 2\pi$
$0 \le x \le 2$





77. $-180° \le x \le -180° + 720°$ One cycle is between the phase shift and phase shift plus period.
$0° \le x + 180° \le 720°$ Add 180° so the left member is 0°.
$0° \le \frac{x}{2} + 90° \le 360°$ Divide each member by 2 so the right member becomes 360°.

$y = 6\sin\left(\frac{x}{2} + 90°\right)$

81. See below.

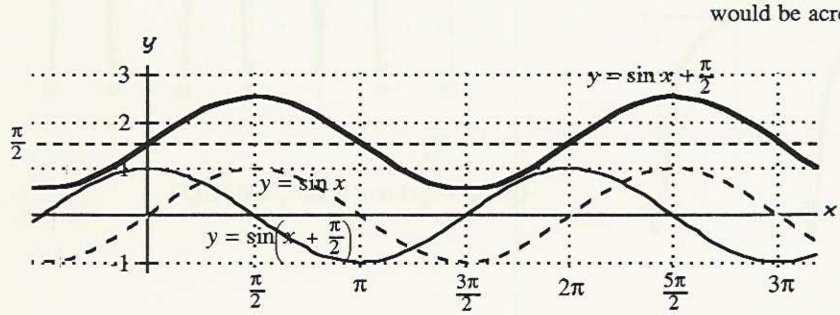
85. $f(x) = 3x^2$
$f(-x) = 3(-x)^2 = 3x^2$
Thus $f(-x) = f(x)$, so the function is even. The symmetry would be about the y-axis.

89. $f(x) = 3\sin x$
$f(-x) = 3\sin(-x) = 3[-\sin x]$
$= -3\sin x$
$-f(x) = -3\sin x$
Thus $f(-x) = -f(x)$, so the function is odd. The symmetry would be across the origin.

93. $f(x) = \frac{\cos x}{x}$
$f(-x) = \frac{\cos(-x)}{-x} = \frac{\cos x}{-x} = -\frac{\cos x}{x}$
$-f(x) = -\frac{\cos x}{x}$
Thus $f(-x) = -f(x)$, so the function is odd. The symmetry would be across the origin.

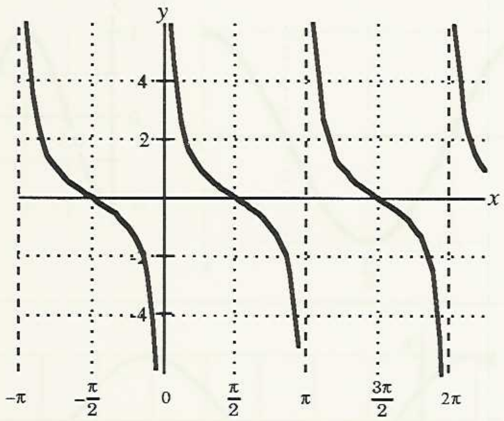
97. $f(x) = \sin^3 x = [\sin x]^3$
$f(-x) = [\sin(-x)]^3 = [-\sin x]^3$
$= -[\sin x]^3 = -\sin^3 x$
$-f(x) = -\sin^3 x$
Thus $f(-x) = -f(x)$, so the function is odd. The symmetry would be across the origin.

81.



Exercise 6-3

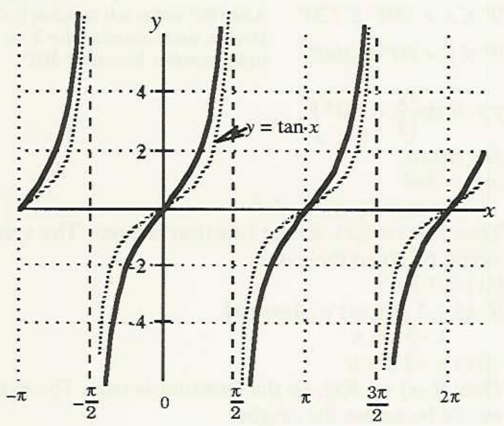
1. Using figure 6.22b we obtain the following graph of the cotangent function.



Figures 6.23 and 6.24 are the graphs of the cosecant and secant functions.

5. $\cot(-x) = \dfrac{1}{\tan(-x)} = \dfrac{1}{-\tan x} = -\dfrac{1}{\tan x} = -\cot x.$

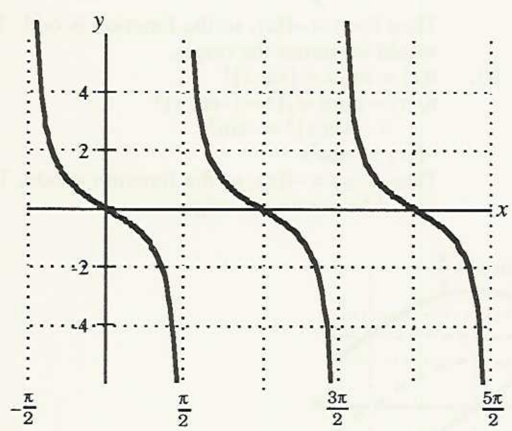
9. $y = 2 \tan x$



13. $y = \cot\left(x - \frac{\pi}{2}\right)$

$0 \le x - \frac{\pi}{2} \le \pi$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ Basic cycle.

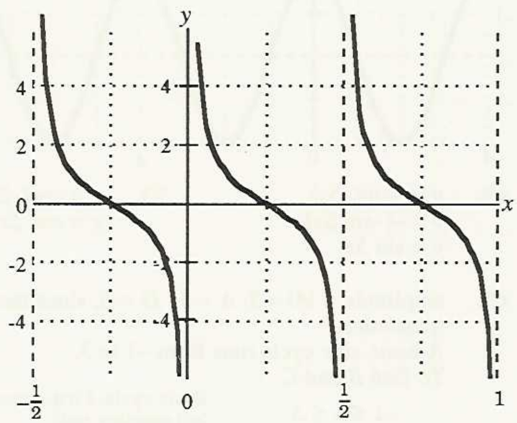


17. $y = \cot 2\pi x$

$0 \le 2\pi x \le \pi$

$0 \le x \le \dfrac{\pi}{2\pi}$

$0 \le x \le \frac{1}{2}$



21. $y = -\cot(-\pi x)$

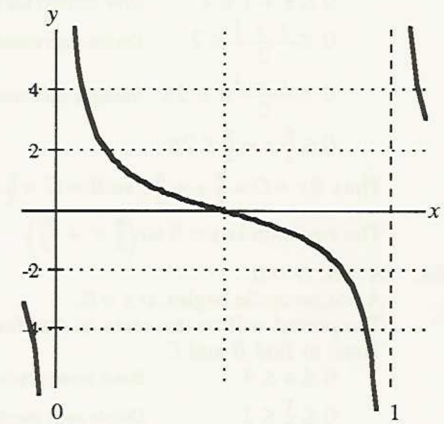
$= -[-\cot \pi x]$

$= \cot \pi x$

$0 \le \pi x \le \pi$

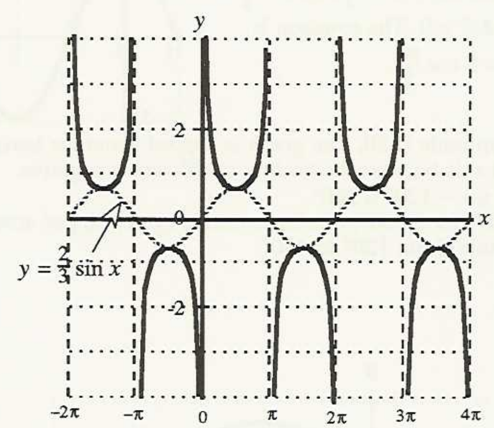
$0 \le x \le 1$

Divide each term by π.



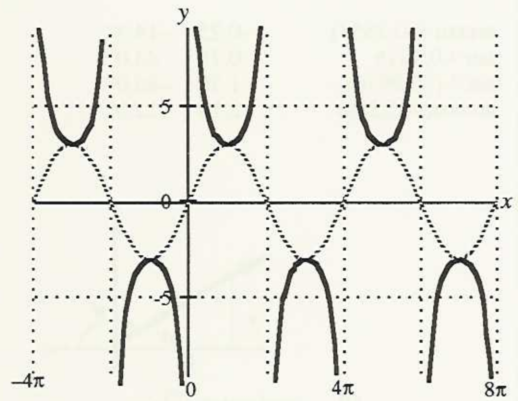
25. $y = \frac{2}{3} \csc x$

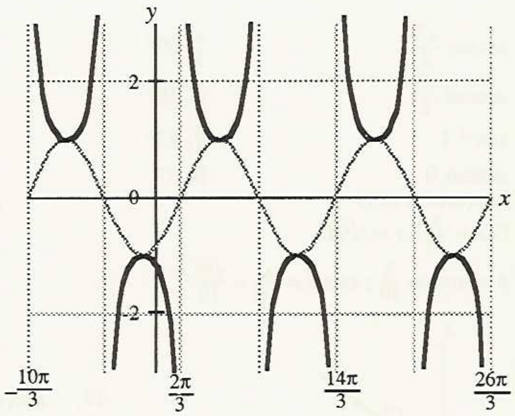
Graph 3 cycles of $y = \frac{2}{3} \sin x$, then "flip the graph".



29. $y = 3 \csc \dfrac{x}{2}$

Graph 3 cycles of $3 \sin \frac{x}{2}$ and "flip".





33. $y = \csc\left(\frac{x}{2} - \frac{\pi}{3}\right)$

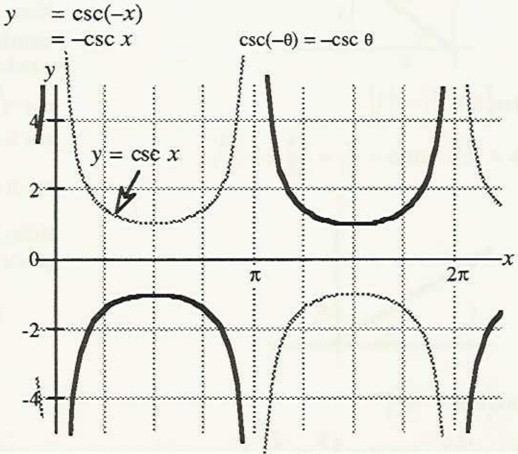
$0 \le \frac{x}{2} - \frac{\pi}{3} \le 2\pi$

$\frac{\pi}{3} \le \frac{x}{2} \le \frac{7\pi}{3}$

$\frac{2\pi}{3} \le x \le \frac{14\pi}{3}$

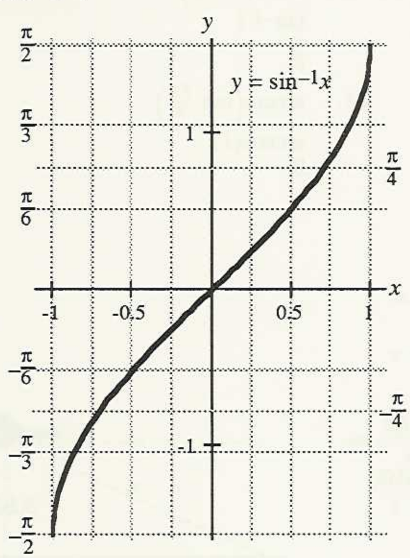
Graph 3 cycles of $y = \sin\left(\frac{x}{2} - \frac{\pi}{3}\right)$.

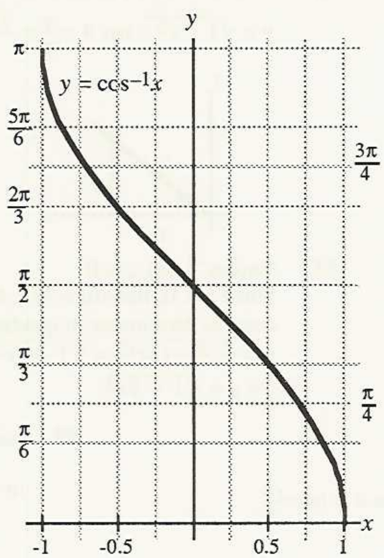
37. $y = \csc(-x)$
$= -\csc x$

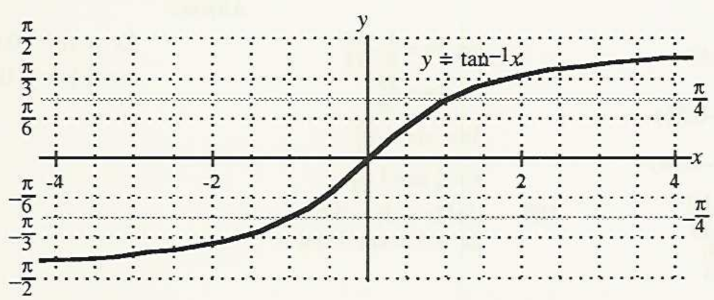


Exercise 6–4

1.







5. $\arcsin \frac{\sqrt{3}}{2}$ $\quad \frac{\pi}{3}$, 60°

9. $\arccos \frac{\sqrt{3}}{2}$ $\quad \frac{\pi}{6}$, 30°

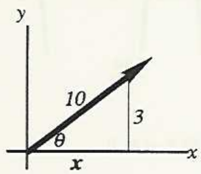
13. $\tan^{-1} 1$ $\quad \frac{\pi}{4}$, 45°

17. $\arctan 0$ $\quad 0$, 0°

37. $\cos(\arcsin 0.3)$

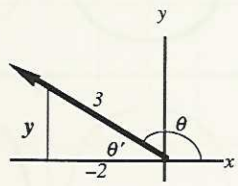
$0.3 = \frac{3}{10}$; $x = \sqrt{91}$

$\theta = \arcsin \frac{3}{10}$; $\cos \theta = \frac{x}{10} = \frac{\sqrt{91}}{10}$



45. $\tan\left[\cos^{-1}\left(-\frac{2}{3}\right)\right]$

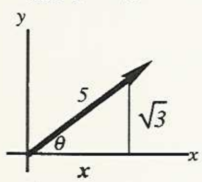
$y = \sqrt{5}$; $\tan \theta = \frac{y}{-2} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$.



49. $\cot\left(\sin^{-1} \frac{\sqrt{3}}{5}\right)$

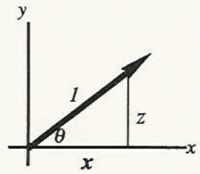
$x = \sqrt{22}$; $\tan \theta = \frac{\sqrt{3}}{x} = \frac{\sqrt{3}}{\sqrt{22}}$;

$\cot \theta = \frac{1}{\tan \theta} = \frac{\sqrt{22}}{\sqrt{3}} = \frac{\sqrt{66}}{3}$.



53. $\cos(\sin^{-1} z)$, $z > 0$

$x = \sqrt{1 - z^2}$; $\cos \theta = \frac{x}{1} = \sqrt{1 - z^2}$.



57. $\sec(\arcsin \sqrt{2z})$

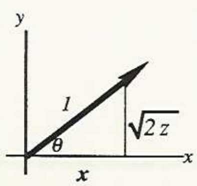
Note that $\sqrt{2z} \geq 0$, so $\theta = \arcsin \sqrt{2z}$ terminates in quadrant I (or is quadrantal).

$x = \sqrt{1^2 - (\sqrt{2z})^2} = \sqrt{1 - 2z}$.

$\cos \theta = \frac{x}{1} = x = \sqrt{1 - 2z}$;

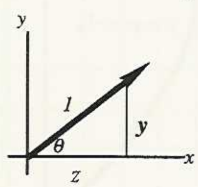
$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\sqrt{1 - 2z}}$.

Note: If $2z = 0$ the angle is quadrantal, but the result is still valid.



61. $\tan(\arccos z)$, $z > 0$

$y = \sqrt{1 - z^2}$; $\tan \theta = \frac{y}{z} = \frac{\sqrt{1 - z^2}}{z}$.

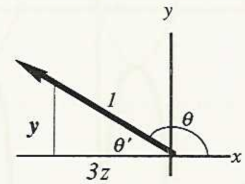


65. $\sin(\cos^{-1} 3z)$, $z < 0$

Since $z < 0$, therefore $3z < 0$, and $\theta = \cos^{-1} 3z$ terminates in quadrant II.

$y = \sqrt{1^2 - (3z)^2} = \sqrt{1 - 9z^2}$; $\sin \theta = \frac{y}{1} = y = \sqrt{1 - 9z^2}$.

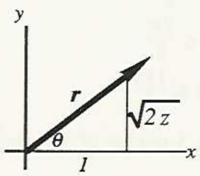
21.	arctan (−0.2553)	−0.25	−14.3°
25.	$\tan^{-1} 0.9316$	0.75	43.0°
29.	$\sin^{-1} (-0.9976)$	−1.50	−86.0°
33.	arccos(−0.5299)	2.13	122.0°



69. $\cos(\arctan \sqrt{2z})$

$r = \sqrt{1^2 + (\sqrt{2z})^2} = \sqrt{1 + 2z}$.

$\cos \theta = \frac{1}{r} = \frac{1}{\sqrt{1 + 2z}}$.



73. $\arcsin\left(\cos \frac{2\pi}{3}\right)$

$\arcsin(-\frac{1}{2})$

$-\arcsin \frac{1}{2}$

$-\frac{\pi}{6}$

77. $\tan^{-1}\left(\sin \frac{\pi}{2}\right)$

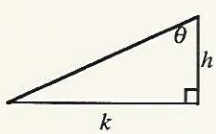
$\tan^{-1} 1$

$\frac{\pi}{4}$

81. $\arccos\left(\tan \frac{5\pi}{4}\right)$

arccos(1)

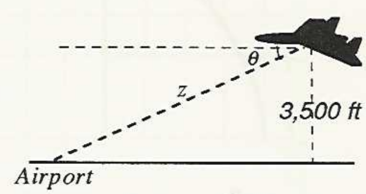
0

85. 

$\tan \theta = \frac{k}{h}$, so $\theta = \tan^{-1}\frac{k}{h}$.

89. $\sin \theta = \frac{3500}{z}$, so

$\theta = \sin^{-1}\frac{3500}{z}$.



93. $\tan \theta = 4.1$

$\theta = \tan^{-1} 4.1$

97. $\frac{\tan \theta}{5} = 10$

$\tan \theta = 50$

$\theta = \tan^{-1} 50$

101. $\sin \frac{3\theta}{2} = -0.56$

$\frac{3\theta}{2} = \sin^{-1}(-0.56)$

$\theta = \frac{2}{3} \sin^{-1}(-0.56)$

105. $\frac{6 \sin 5\theta}{5} = \frac{10}{13}$

$6 \sin 5\theta = \frac{50}{13}$

$\sin 5\theta = \frac{1}{6} \cdot \frac{50}{13}$

$\sin 5\theta = \frac{25}{39}$

$5\theta = \sin^{-1} \frac{25}{39}$

$\theta = \frac{1}{5} \sin^{-1} \frac{25}{39}$

109. $\sin(2x + 3) = 0.6$

$2x + 3 = \sin^{-1} 0.6$

$2x = \sin^{-1} 0.6 - 3$

$x = \frac{1}{2} (\sin^{-1} 0.6 - 3)$

Exercise 6–5

1. $\csc^{-1} 2 = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

5. $\sec^{-1}(-2) = \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

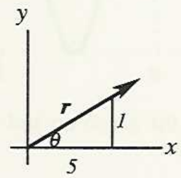
9. $\operatorname{arccot} 0 = \frac{\pi}{2} - \arctan 0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$.

13. $\operatorname{arcsec}(-2.9986) = \cos^{-1}\left(-\frac{1}{2.9986}\right) \approx 1.91$
$\approx 109.5°$

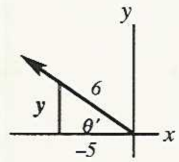
17. $\sec^{-1}(-11.1261) = \cos^{-1}\left(-\frac{1}{11.1261}\right) \approx 1.66$
$\approx 95.2°$

21. $\sin(\csc^{-1} 3) = \sin(\sin^{-1}\frac{1}{3}) = \frac{1}{3}$

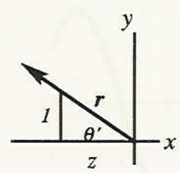
25. csc(arccot 5) If θ = arccot 5, then $\cot\theta = 5$, and $\tan\theta = \frac{1}{5}$. $r = \sqrt{26}$;
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{r}} = r = \sqrt{26}$.



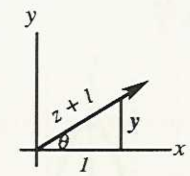
29. $\tan\left[\sec^{-1}\left(-\frac{6}{5}\right)\right] = \tan\left[\cos^{-1}\left(-\frac{5}{6}\right)\right]$
$y = \sqrt{11}$; $\tan\theta = \frac{y}{-5} = -\frac{\sqrt{11}}{5}$.



33. $\sec(\cot^{-1}z)$, $z < 0$; If $\theta = \cot^{-1}z$, $z < 0$, then θ is an angle terminating in quadrant II, and $\tan\theta = \frac{1}{z}$.
$r = \sqrt{z^2 + 1}$;
$\sec\theta = \frac{1}{\cos\theta}$
$= \frac{1}{\frac{z}{r}} = \frac{r}{z}$
$= \frac{\sqrt{z^2 + 1}}{z}$



37. $\tan[\sec^{-1}(z + 1)]$, $z + 1 > 0 =$
$\tan\left(\cos^{-1}\frac{1}{z + 1}\right)$
$y = \sqrt{(z + 1)^2 - 1^2} = \sqrt{z^2 + 2z}$; $\tan\theta = \frac{y}{1} = y = \sqrt{z^2 + 2z}$.



41. 0.216 (0.2160201274583045647)

Chapter 6 Review

$\frac{\theta°}{180°} = \frac{s}{\pi}$, so $s = \frac{\theta°\pi}{180°}$.

1. 315° $s = \frac{\theta°\pi}{180°} = \frac{315°\pi}{180°} = \frac{7}{4}\pi \approx$ 5.50

3. –148° $s = \frac{\theta°\pi}{180°} = \frac{-148°\pi}{180°} = -\frac{37}{45}\pi \approx$ –2.58

$\frac{\theta°}{180°} = \frac{s}{\pi}$, so $\theta° = \frac{180°}{\pi}s$

5. $\frac{7\pi}{9}$ $\theta° = \frac{180°}{\pi}s = \frac{180°}{\pi} \cdot \frac{7\pi}{9} = 140°$

7. –4 $\theta° = \frac{180°}{\pi}s = \frac{180°}{\pi} \cdot (-4) = -\frac{720}{\pi}° \approx$ –229.18°

9. $L = rs$
$r = \frac{1}{2}$ diameter = 8"
$L = 8(3.8) = 30.4"$

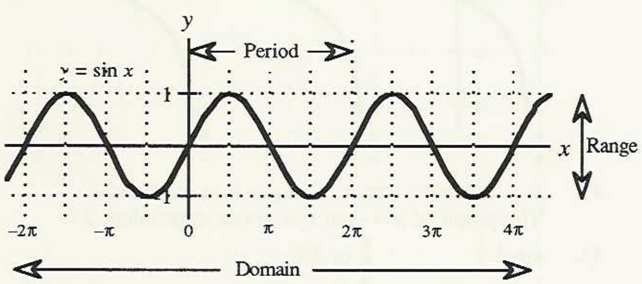
11. $r = \frac{1}{2}$ diameter = 10"
$L = rs$
$32 = 10s$
$s = \frac{32}{10} = \frac{16}{5} = 3\frac{1}{5}$ (radians)
$\frac{16}{5}$ radians is $\frac{180°}{\pi} \cdot \frac{16}{5} \approx 183.3°$.

Make sure the calculator is in radian mode.

13. $\sin 4 \approx -0.7568$

15. $\sec\frac{3}{8} = \frac{1}{\cos\frac{3}{8}} \approx 1.0747$

17. $\cos\frac{5\pi}{6}$; $\frac{5\pi}{6}$ terminates in qII, where the cosine function is negative; also θ' $= \pi - \frac{5\pi}{6} = \frac{\pi}{6}$. $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.
Thus $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$.

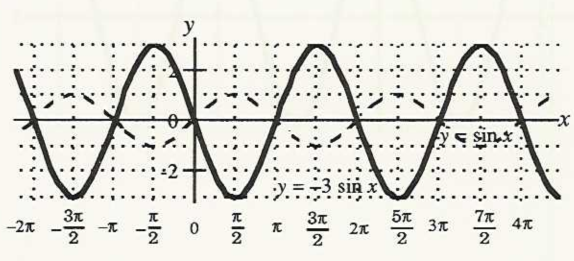


19. Domain: R; Range: $-1 \le y \le 1$; Period: 2π.

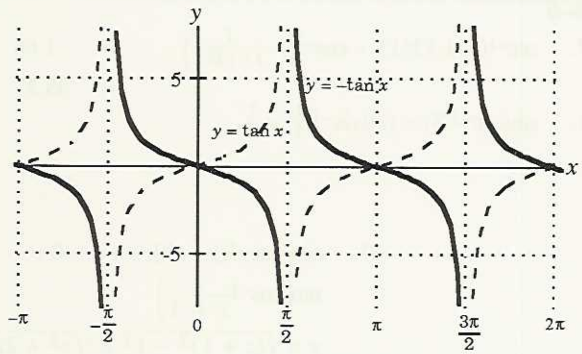
21. $\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$ sine is an odd function, so $\sin(-x) = -\sin x$

23. $f(x) = x \sin x$
$f(-x) = (-x)\sin(-x)$
$= (-x)(-\sin x)$ $\sin(-\theta) = -\sin\theta$.
$= x \sin x$
$= f(x)$.
f is an even function, since $f(-x) = -f(x)$. It would exhibit symmetry about the y-axis.

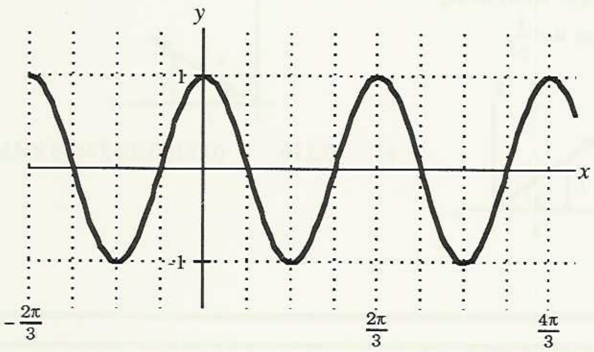
25. $y = -3 \sin x$



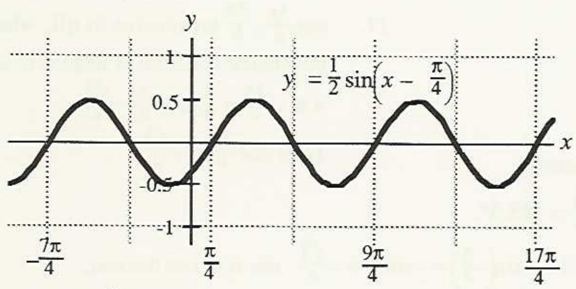
27. $y = -\tan x$



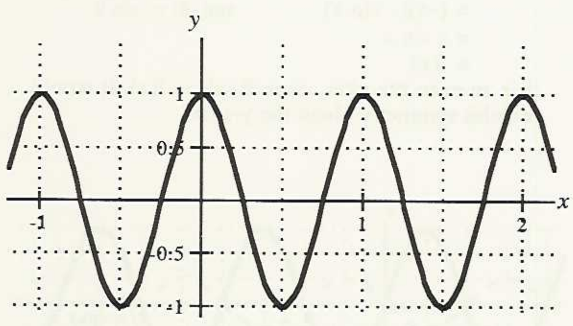
29. $y = \cos 3x$
$0 \le 3x \le 2\pi$
$0 \le x \le \frac{2\pi}{3}$



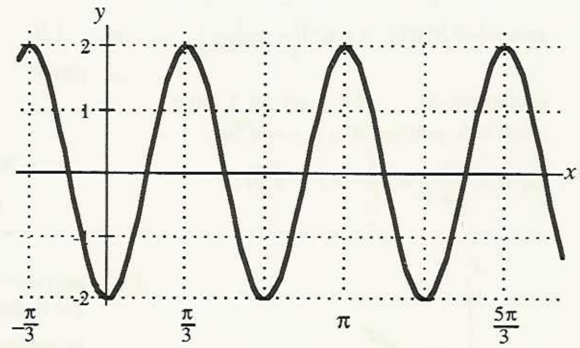
31. $y = \frac{1}{2}\sin\left(x - \frac{\pi}{4}\right)$
$0 \le x - \frac{\pi}{4} \le 2\pi$
$\frac{\pi}{4} \le x \le \frac{5\pi}{4}$



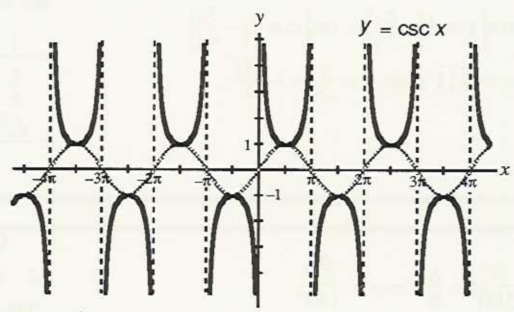
33. $y = \cos 2\pi x$
$0 \le 2\pi x \le 2\pi$
$0 \le x \le 1$



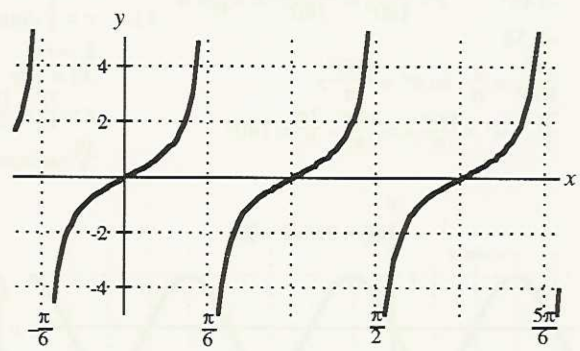
35. $y = 2\cos(\pi - 3x)$ Observe that the coefficient of x is negative.
$= 2\cos[-(3x - \pi)]$ $b - a = -(a - b)$.
$= 2\cos(3x - \pi)$ $\cos(-\theta) = \cos\theta$.
$0 \le 3x - \pi \le 2\pi$
$\pi \le 3x \le 3\pi$
$\frac{\pi}{3} \le x \le \pi$



37. Graph $y = \sin x$, then "fold" the graph up and down, starting at its highest and lowest points.



39. $y = \tan 3x$
$-\frac{\pi}{2} \le 3x \le \frac{\pi}{2}$ Basic tangent cycle.
$-\frac{\pi}{6} \le x \le \frac{\pi}{6}$ Multiply each member by $\frac{1}{3}$.



41. $y = \tan(-x) = -\tan x$ tangent is an odd function.
The graph of $y = -\tan x$ is given in problem 27.

43. $\sin^{-1}\frac{1}{2}$ $\frac{\pi}{6}$ or 30°

45. $\arctan\sqrt{3}$ $\frac{\pi}{3}$ or 60°

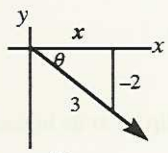
47. arctan (–3.55) –1.30 (radians) or –74.3°

49. $\tan^{-1} 2.5$ 1.19 or 68.2°

51. $\tan\left[\arcsin\left(-\frac{2}{3}\right)\right]$

$x = \sqrt{5}$; $\tan\theta = \frac{-2}{x}$

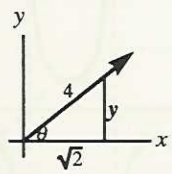
$= -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$



53. $\csc\left(\cos^{-1}\frac{\sqrt{2}}{4}\right)$

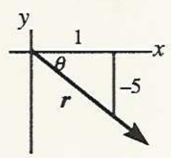
$y = \sqrt{14}$; $\csc\theta = \frac{1}{\sin\theta}$

$= \frac{4}{y} = \frac{4}{\sqrt{14}} = \frac{4\sqrt{14}}{14} = \frac{2\sqrt{14}}{7}$



55. $\sin(\arctan(-5))$

$r = \sqrt{26}$; $\sin\theta = \frac{-5}{r} = -\frac{5}{\sqrt{26}} = -\frac{5\sqrt{26}}{26}$.

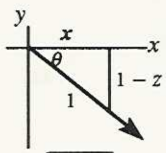


57. $\tan[\sin^{-1}(1 - z)]$, $1 - z < 0$

$x = \sqrt{1^2 - (1 - z)^2}$

$= \sqrt{2z - z^2}$;

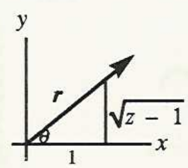
$\tan\theta = \frac{1 - z}{x} = \frac{1 - z}{\sqrt{2z - z^2}}$.



59. $\sec(\tan^{-1}\sqrt{z - 1})$

$r = \sqrt{1^2 + (\sqrt{z - 1})^2} = \sqrt{z}$;

$\sec\theta = \frac{1}{\cos\theta} = r = \sqrt{z}$.



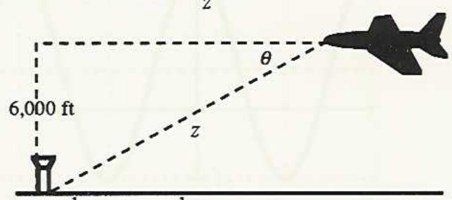
61. $\sin^{-1}\left(\cos\frac{\pi}{2}\right)$

$\sin^{-1} 0$

0

63. If θ represents the angle, $\sin\theta = \frac{6000}{z}$,

so $\theta = \sin^{-1}\frac{6000}{z}$.



65. $\frac{1}{3}\sin 2x = \frac{1}{12}$

$\sin 2x = \frac{1}{4}$

$2x = \sin^{-1}\frac{1}{4}$

$x = \frac{1}{2}\sin^{-1}\frac{1}{4}$.

67. $\csc^{-1} 2 = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

69. $\cot^{-1} 3 = \frac{\pi}{2} - \tan^{-1}3 \approx 0.32$

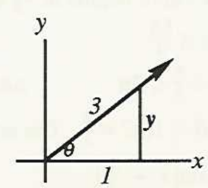
$= 90° - \tan^{-1} 3 \approx 18.4°$

71. $\cot(\sec^{-1} 3)$

If $\theta = \sec^{-1} 3$, then $\sec\theta = 3$, so $\cos\theta = \frac{1}{3}$.

$y = \sqrt{8} = 2\sqrt{2}$; $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{y} = \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{2}}{4}$.



Chapter 6 Test

1. $\frac{\theta°}{180°} = \frac{s}{\pi}$

$\frac{-250°}{180°} = \frac{s}{\pi}$

$s = -\frac{25}{18}\pi \approx -4.36$

3. $r = 10$ inches (half the diameter);

$L = rs$

$L = 10(2.5) = 25$ inches.

5. 5 feet = 5•12 = 60 inches

r = half of 38 = 19;

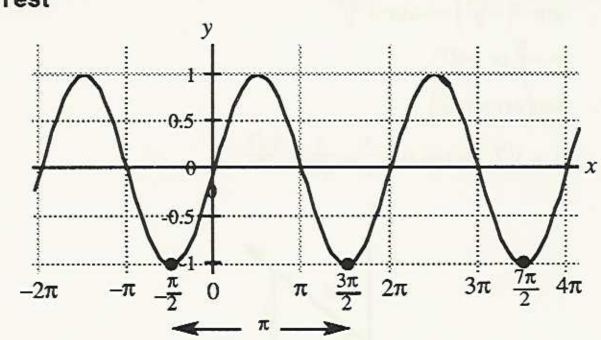
$L = rs$

$60 = 19s$

$s = \frac{60}{19}$; $\frac{\theta°}{360°} = \frac{\frac{60}{19}}{2\pi}$; $\theta° = \frac{10800}{19\pi}° \approx 181°$.

7. $\frac{11\pi}{6}$ terminates in quadrant IV, so $\theta' = 2\pi - \theta = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$, and $\cos\theta > 0$. $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so $\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$.

9. One value for which $\sin x = -1$ is at $\frac{3\pi}{2}$. All the other values are integer multiples of 2π units from this value. Thus the values are $x = \frac{3\pi}{2} + 2k\pi$, k an integer.



11. $f(x) = x + \sin x$

$f(-x) = (-x) + \sin(-x)$

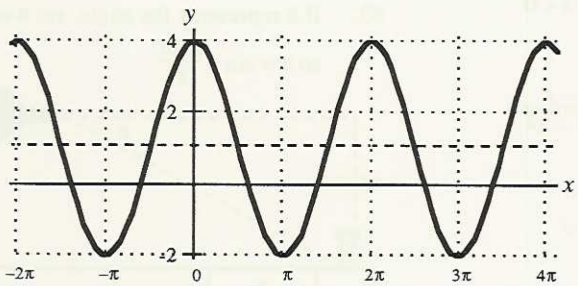
$= (-x) + (-\sin x)$

$= -(x + \sin x)$

$= -f(x)$.

Since $f(-x) = -f(x)$, f is an odd function. It's graph would have symmetry about the origin.

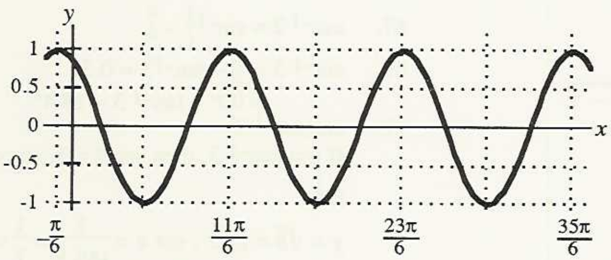
13. $y = 3\cos x + 1$



15. $y = \cos\left(x + \frac{\pi}{6}\right)$

$0 \le x + \frac{\pi}{6} \le 2\pi$

$-\frac{\pi}{6} \le x \le \frac{11\pi}{6}$



17. $A = +2$, and $D = 0$.

A sine curve begins at $-\frac{\pi}{3}$ and ends at $\frac{5\pi}{3}$.

$-\frac{\pi}{3} \le x \le \frac{5\pi}{3}$

$0 \le x + \frac{\pi}{3} \le 2\pi$ Add $\frac{\pi}{3}$ to each member.

Thus $B = 1$, $C = \frac{\pi}{3}$. The equation is

$y = 2\sin\left(x + \frac{\pi}{3}\right)$.

19. $f(x) = \sec x \cdot \sin x + x^3$

$f(-x) = \sec(-x) \cdot \sin(-x) + (-x)^3$

$= \sec x \cdot (-\sin x) + (-x^3)$ secant is even, sine is odd, x^3 is odd.

$= -\sec x \cdot \sin x - x^3$

$= -(\sec x \cdot \sin x + x^3)$

$= -f(x)$.

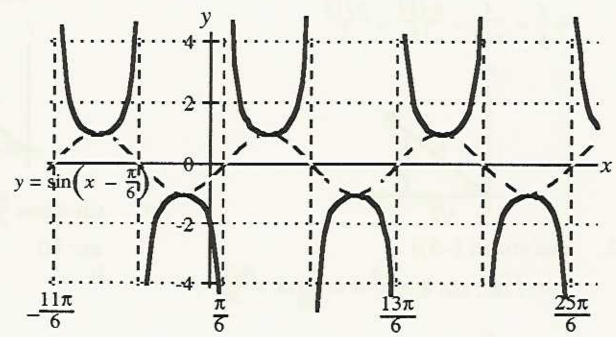
Thus, $f(x)$ is an odd function.

21. $y = \csc\left(x - \frac{\pi}{6}\right)$

Graph $y = \sin\left(x - \frac{\pi}{6}\right)$ and "flip" it at its highest and lowest points.

$0 \le x - \frac{\pi}{6} \le 2\pi$

$\frac{\pi}{6} \le x \le \frac{13\pi}{6}$

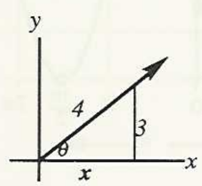


23. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\frac{\sqrt{3}}{2}$

$= -\frac{\pi}{3}$ or $-60°$.

25. $\tan\left(\arcsin\frac{3}{4}\right)$

$x = \sqrt{7}$, so $\tan\theta = \frac{3}{x} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$



0

27. $\csc\left(\sin^{-1}\frac{\sqrt{3}}{4}\right)$

If $\theta = \sin^{-1}\frac{\sqrt{3}}{4}$, then $\sin\theta = \frac{\sqrt{3}}{4}$, and $\csc\theta = \frac{1}{\sin\theta}$

$= \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$.

29. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

31. $\cos\theta = \frac{z}{5}$, so $\theta = \cos^{-1}\frac{z}{5}$.

33. $\sec^{-1}2.65 = \cos^{-1}\frac{1}{2.65} \approx 1.18$ (radians) or $67.8°$